

**Polytropic Head for a Centrifugal Compressor****SI Units:**

$$H_{poly} = \frac{8314}{MW} \times T_1 \times Z_{avg} \times \left( \frac{n}{n-1} \right) \times \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

where:

- $H_{poly}$  = Polytropic head, N-m/kg  
 MW = molecular weight of the gas, kg/kg-mol  
 $T_1$  = Absolute temperature at inlet conditions, K  
 $Z_{avg}$  = Average compressibility factor, dimensionless  
           =  $(Z_1 + Z_2) / 2$   
 $Z_1$  = Compressibility factor at inlet conditions, dimensionless  
 $Z_2$  = Compressibility factor at discharge conditions, dimensionless  
 $P_1$  = Inlet pressure in absolute units, kPaa or bara  
 $P_2$  = Discharge pressure in absolute units, kPaa or bara  
 n = polytropic exponent

**US Customary units**

$$H_{poly} = \frac{1545}{MW} \times T_1 \times Z_{avg} \times \left( \frac{n}{n-1} \right) \times \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

- $H_{poly}$  = Polytropic head, ft-lb<sub>f</sub>/lb<sub>m</sub>  
 MW = molecular weight of the gas, lb/lb-mol  
 $T_1$  = Absolute temperature at inlet conditions, R  
 $Z_{avg}$  = Average compressibility factor, dimensionless  
           =  $(Z_1 + Z_2) / 2$   
 $Z_1$  = Compressibility factor at inlet conditions, dimensionless  
 $Z_2$  = Compressibility factor at discharge conditions, dimensionless  
 $P_1$  = Inlet pressure in absolute units, psia  
 $P_2$  = Discharge pressure in absolute units, psia

Revision: 0

Prepared By:

Checked By:

Approved By

**Centrifugal Compressor Performance****SI Units****Impeller Flow Coefficients**

$$\phi = \frac{V_{inlet}}{u_2 \times d_2^2}$$

where:

 $\phi$  = impeller flow coefficient, dimensionless $V_{inlet}$  = compressor inlet volume flow in m<sup>3</sup>/s $u_2$  = impeller tip speed in m/s $d_2$  = impeller diameter in m**Note:** Impeller flow coefficients shall be within the 0.01 to 0.12 range**Impeller Stage Head Coefficients****SI Units**

$$\mu = \frac{H_{poly}}{u_2^2}$$

where:

 $\mu$  = Head Coefficient, dimensionless $H_{poly}$  = Polytropic head **per stage**, N-m/kg**Note:** Impeller stage head coefficients shall be within the 0.4 to 0.55 range. For preliminary calculations an initial value of 0.48 is recommended.**Approximate Rotational Speed****SI Units**

$$N = \frac{u_2 \times 60}{\pi \times d_2}$$

where:

 $N$  = approximate rotational speed, rpm**US Cust Units**

$$\phi = \frac{V_{inlet} \times 144}{u_2 \times d_2^2}$$

 $\phi$  = impeller flow coefficient, dimensionless $V_{inlet}$  = compressor inlet volume flow in ft<sup>3</sup>/s $u_2$  = impeller tip speed in ft/s $d_2$  = impeller diameter in inch**US Cust Units**

$$\mu = \frac{H_{poly} \times 32.2}{u_2^2}$$

 $\mu$  = Head Coefficient, dimensionless $H_{poly}$  = Polytropic head **per stage**, ft-lb<sub>f</sub>/lb<sub>m</sub>**US Cust Units**

$$N = \frac{u_2 \times 60 \times 12}{\pi \times d_2}$$

 $N$  = approximate rotational speed, rpm

Revision: 0

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Discharge compressibility factor,  $Z_2 =$  0.941 (Calculated from "Comp. Factor SI" worksheet)  
Average Compressibility,  $Z_{avg} =$  0.956 ( $(Z_1+Z_2)/2$ )

Average  $\kappa$ -value  
The inlet  $\kappa$  value is generally used for polytropic head calculations. The polytropic head equation is insensitive to  $\kappa$ -value (and therefore  $n$ -value) within the limits that  $\kappa$  normally varies during compression. This is because any multiplier in the polytropic head equation tends to balance corresponding errors in the  $(n-1)/n$  exponent.

Thus,  
 $\kappa_{avg} = \kappa_1 =$  1.126

Polytropic Head,  
 $H_{poly} =$  71,971 N-m/kg (For equation refer "Head eqn" sheet)

**Number of Stages**

**Definition of Stage :** For the purpose of this spreadsheet, a stage is defined as an impeller-diffuser pair. A series of stages will comprise a 'section' or 'process stage' between which there is no intercooling or loss or gain in flow (i.e. no sidestreams, bypassing or injection) & whose rotational speeds are in fixed ratio to each other.

**DEP 31.29.40.10-Gen. :** "Single section compressors with in-line impeller arrangements shall have no more than 9 impellers per casing. Two section compressors shall have no more than 8 impellers per casing. Compressors with one side-stream shall have a maximum of 7 impellers per casing, for each additional sidestream one additional impeller less in order to have sufficient space for adequate mixing of main & side streams."

Thus number of stages = 2.9 ( $H_{poly}/H_{poly-nom}$ )  
3 (Round-up to the next whole integer if calculated stages  $>X.2$  & round-down if  $\leq X.2$ , where  $X$  = a whole integer)

Calculated polytropic head per stage = 23,990 N-m/kg

**Rotational Speed**

$N_{nom} =$  5,900 rpm  
Actual rotational speed = 5,754 rpm ( $N_{nom} * ((H_{poly}/H_{poly-nom} * \text{No. of Stages})^{0.5})$ )

**Shaft Power,  $P_{shaft}$**

Gas Power Requirement,  $P_{gas} =$  3,533.1 kW ( $M * H_{poly} / (3.6E+6 * \eta_{poly})$ )

**Table 2**  
**Approximate Mechanical Losses as a Percentage of Gas Power Requirement**

Gas Power Requirement		Mechanical Losses, $L_m$ ,
English (hp)	Metric (kW)	%
0-3,000	0-2,500	3
3,000-6,000	2,500-5,000	2.5
6,000-10,000	5,000-7,500	2
10,000+	7,500+	1.5

(Table reproduced from page 236 of "Pipeline Rules of Thumb Handbook" by E.W. McAllister, 3rd Ed., Gulf Publishing Company, Houston, TX)

Mechanical Losses for given example:  
 $L_m =$  2.5 % (Table 2)  
88.3 kW

Shaft power requirement is sum of gas power requirement and mechanical losses. Thus  
 $P_{shaft} =$  3,621.4 kW  
3,622 kW

**Summary of Results:**

Polytropic Head, 71,971 N-m/kg  
 $H_{poly} =$   
No. of Stages = 3  
Discharge Temperature,  $T_2$  99.9 °C  
Rotational Speed = 5,754 rpm  
Shaft Power,  $P_{shaft} =$  3,622.0 kW  
=

Data Table reproduced from: "GPSA Engineering Data Book, 11th Edition-SI, 1998"

Component	Critical Pressure, kPa (abs)	Critical Temperature, K	Molecular Weight	Specific Heat at constant pressure, $C_p$ , KJ/kg-K
Methane	4,599	190.56	16.043	2.204
Ethane	4,880	305.41	30.07	1.7053
Propane	4,240	369.77	44.097	1.6242
Isobutane	3,640	407.82	58.123	1.6164
n-Butane	3,784	425.1	58.123	1.6514
Isopentane	3,381	460.35	72.15	1.6071
n-Pentane	3,365	469.65	72.15	1.6233
Neopentane	3,199	433.71	72.15	1.632
n-Hexane	3,030	506.4	86.177	1.6151
n-Heptane	2,740	539.2	100.204	1.6074
n-Octane	2,490	568.4	114.231	1.6026
n-Nonane	2,280	594.7	128.258	1.599
n-Decane	2,100	617.7	142.285	1.5962
Carbon Dioxide	7,374	304.11	44.01	0.83294
Hydrogen Sulfide	8,963	373.37	34.082	0.99794
Oxygen	5,043	154.59	32	0.91673
Nitrogen	3,398	126.21	28.0134	1.0395
Water	22,064	647.1	18.01	1.8617
Sulfur Dioxide	7,884	430.8	64.065	0.61942
Air	3,771	132.43	28.9625	1.004

Prepared by: **Ankur Srivastava**  
**Chemical Engineer**  
E-mail:  
[ankur\\_2061@hotmail.com](mailto:ankur_2061@hotmail.com)



Discharge compressibility factor,  $Z_2 =$  0.941 (Calculated from "Comp. Factor US" worksheet)  
 Average Compressibility,  $Z_{avg} =$  0.956 ( $(Z_1+Z_2)/2$ )

Average  $\kappa$ -value  
 The inlet  $\kappa$  value is generally used for polytropic head calculations. The polytropic head equation is insensitive to  $\kappa$ -value (and therefore  $n$ -value) within the limits that  $\kappa$  normally varies during compression. This is because any multiplier in the polytropic head equation tends to balance corresponding errors in the  $(n-1)/n$  exponent.

Thus,  
 $\kappa_{avg} = \kappa_1 =$  1.125

Polytropic Head,  
 $H_{poly} =$  24,089 ft-lb<sub>f</sub> / lb<sub>m</sub> (For equation refer Head eqn sheet)

**Number of Stages**

**Definition of Stage :** For the purpose of this spreadsheet, a stage is defined as an impeller-diffuser pair. A series of stages will comprise a 'section' or 'process stage' between which there is no intercooling or loss or gain in flow (i.e. no sidestreams, bypassing or injection) & whose rotational speeds are in fixed ratio to each other.

**DEP 31.29.40.10-Gen. :** "Single section compressors with in-line impeller arrangements shall have no more than 9 impellers per casing. Two section compressors shall have no more than 8 impellers per casing. Compressors with one side-stream shall have a maximum of 7 impellers per casing, for each additional sidestream one additional impeller less in order to have sufficient space for adequate mixing of main & side streams."

Thus number of stages = 2.9 ( $H_{poly}/H_{poly-nom}$ )  
3 (Round-up to the next whole integer if calculated stages >X.2 & round-down if  $\leq X.2$ , where X = a whole integer)

Calculated polytropic head per stage = 8,030 ft-lb<sub>f</sub> / lb<sub>m</sub>

**Rotational Speed**

$N_{nom} =$  5,900 rpm  
 Actual rotational speed = 5,754 rpm ( $N_{nom} * ((H_{poly}/H_{poly-nom})^{0.5})$ )

**Shaft Power, P<sub>shaft</sub>**

Gas Power Requirement,  $P_{gas} =$  4,740.0 hp ( $M * H_{poly} / (1.98E+6 * \eta_{poly})$ )

**Table 2**  
**Approximate Mechanical Losses as a Percentage of Gas Power Requirement**

Gas Power Requirement		Mechanical Losses, $L_m$ ,
English (hp)	Metric (kW)	%
0-3,000	0-2,500	3
3,000-6,000	2,500-5,000	2.5
6,000-10,000	5,000-7,500	2
10,000+	7,500+	1.5

(Table reproduced from page 236 of "Pipeline Rules of Thumb Handbook" by E.W. McAllister, 3rd Ed., Gulf Publishing Company, Houston, TX)

Mechanical Losses for given example:  
 $L_m =$  2.5 % (Table 2)  
118.5 hp  
 Shaft power requirement is sum of gas power requirement and mechanical losses. Thus  
 $P_{shaft} =$  4,858.5 hp  
4,859 hp

**Summary of Results:**

Polytropic Head, 24,089 ft-lb<sub>f</sub> / lb<sub>m</sub>  
 $H_{poly} =$   
 No. of Stages = 3  
 Discharge Temperature,  $T_2 =$  211.4 °F  
 Rotational Speed = 5,754 rpm  
 Shaft Power,  $P_{shaft} =$  4,859.0 hp  
 =

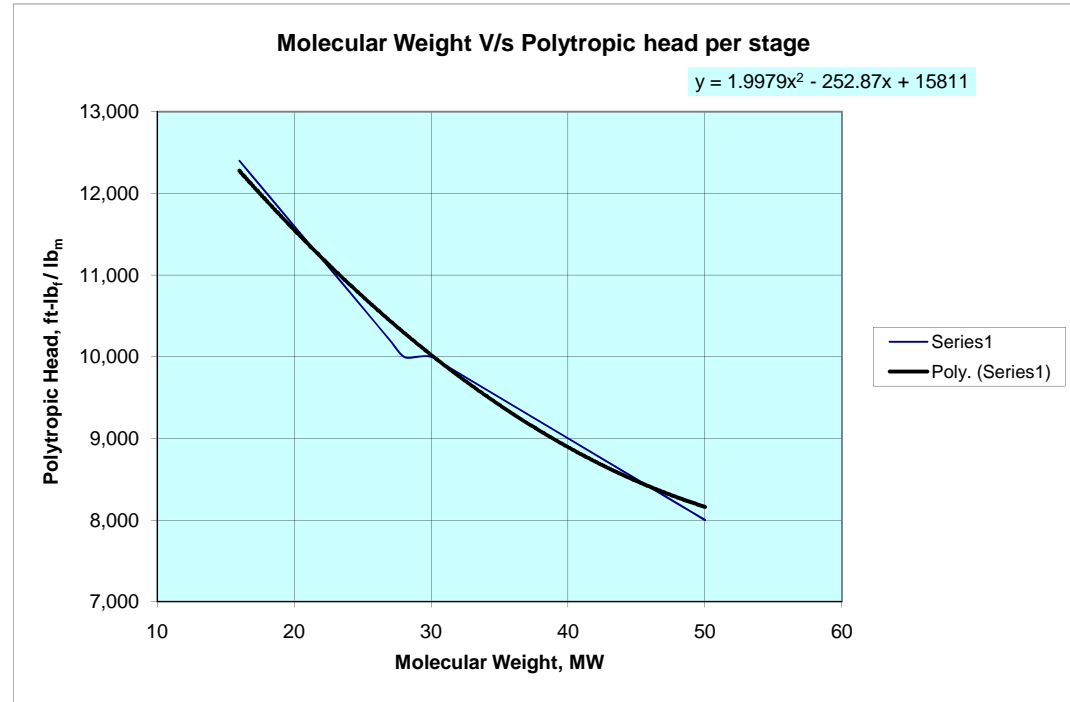
Data Table reproduced from: "GPSA Engineering Data Book, 12th Edition-FPS, 2004"

Component	Critical Pressure, psia	Critical Temperature, °F	Molecular Weight	Specific Heat at constant pressure, C <sub>p</sub> , Btu/lb-°F
Methane	667	-116.66	16.042	0.52725
Ethane	706.6	89.92	30.069	0.4088
Propane	615.5	205.92	44.096	0.3897
Isobutane	527.9	274.41	58.122	0.39709
n-Butane	550.9	305.55	58.122	0.39649
Isopentane	490.4	369	72.149	0.38311
n-Pentane	488.8	385.8	72.149	0.38882
Neopentane	463.5	321	72.149	0.39126
n-Hexane	436.9	453.8	86.175	0.38696
n-Heptane	396.8	512.9	100.202	0.38566
n-Octane	360.7	564.2	114.229	0.38446
n-Nonane	330.7	610.8	128.255	0.38358
n-Decane	304.6	652.2	142.282	0.38294
Carbon Dioxide	1,070	87.76	44.01	0.19875
Hydrogen Sulfide	1,306.5	212.81	34.082	0.23811
Oxygen	731.4	-181.43	31.9988	0.21894
Nitrogen	492.5	-232.53	28.0135	0.24829
Water	3,200.1	705.1	18.0153	0.44476
Sulfur Dioxide	1,143	315.48	64.065	0.14735
Air	551.9	-220.97	28.9586	0.23992

Prepared by: **Ankur Srivastava**  
**Chemical Engineer**  
**E-mail:**  
[ankur\\_2061@hotmail.com](mailto:ankur_2061@hotmail.com)



Molecular Weight, MW	Polytropic head per stage, ft-lb/lb <sub>m</sub>	Calculated head per eqn., ft-lb/lb <sub>m</sub>	Error % for tabulated and eqn. values
16	12,400	12,277	-1.00
17	12,200	12,090	-0.90
18	12,000	11,907	-0.78
19	11,800	11,728	-0.61
20	11,600	11,553	-0.41
21	11,400	11,382	-0.16
22	11,200	11,215	0.13
23	11,000	11,052	0.47
24	10,800	10,893	0.86
25	10,600	10,738	1.30
26	10,400	10,587	1.80
27	10,200	10,440	2.35
28	10,000	10,297	2.97
29	10,000	10,158	1.58
30	10,000	10,023	0.23
31	9,900	9,892	-0.08
32	9,800	9,765	-0.36
33	9,700	9,642	-0.60
34	9,600	9,523	-0.80
35	9,500	9,408	-0.97
36	9,400	9,297	-1.10
37	9,300	9,190	-1.18
38	9,200	9,087	-1.23
39	9,100	8,988	-1.23
40	9,000	8,893	-1.19
41	8,900	8,802	-1.10
42	8,800	8,715	-0.97
43	8,700	8,632	-0.78
44	8,600	8,553	-0.55
45	8,500	8,478	-0.26
46	8,400	8,407	0.08
47	8,300	8,339	0.48
48	8,200	8,276	0.93
49	8,100	8,217	1.45
50	8,000	8,162	2.03



"To determine the number of stages (using the impeller and diffuser defined as the stage), assume 10,000 ft-lb/lb<sub>m</sub> of head per stage. This value can be used if the molecular weight is in the range of 28 to 30. For other molecular weights, this initial value must be modified. As a rule of thumb, lower the head per stage by 100 ft-lb/lb<sub>m</sub> for each unit increase in molecular weight. Conversely, raise the allowable head per stage 200 ft-lb/lb<sub>m</sub> for a unit decrease in molecular weight. The rule of thumb gives the best results for a molecular weight range of 2 through 70. Because this sizing procedure is being used only to establish the rough size of the compressor, the upper range may be extended with some loss in accuracy." - Page 196, Compressors: Selection and Sizing, 3rd Edition, by Royce N. Brown

Revision: 0  
 Prepared By:  
 Checked By:  
 Approved By:

**This sheet is applied for the "Polytropic Head US Cust" worksheet****Calculations for Inlet Compressibility Factor,  $Z_1$  using Redlich-Kwong Equation of State**

Molecular Weight, M	45.50	g/g-mol
Pseudo-Critical Temp., $T_c$ :	375.04	°K
Pseudo-Critical Press., $P_c$ :	41.53	atmA
Inlet temperature, $T_1$	313.33	°K
Inlet Pressure, $P_1$	2.041	atmA

**Constants Used:**

Gas Constant, R: 0.08206 L\*atm/g-mol\*°K

**Calculated Parameters:**

Reduced Temperature,  $T_r$  0.835 ( $T/T_c$ )  
 Reduced Pressure,  $P_r$  0.049 ( $P/P_c$ )

**Equations:**

$A^2 =$	0.03293
B =	0.00510
q =	-0.02781
r =	0.00017
f =	-0.30553
g =	-0.06497
C =	0.00000
D =	IRRELEVANT
E1 =	IRRELEVANT
$\phi =$	0.02948
E =	IRRELEVANT
z1 =	0.97156
z2 =	0.00879
z3 =	0.01965
z0 =	IRRELEVANT
z =	<b>0.972</b>

**Calculations for Discharge Compressibility Factor,  $Z_2$  using Redlich-Kwong Equation of State**

Molecular Weight, M	45.50	g/g-mol
Pseudo-Critical Temp., $T_c$ :	375.04	°K
Pseudo-Critical Press., $P_c$ :	41.53	atmA
Discharge temperature, $T_2$	372.98	°K
Discharge Pressure, $P_2$	6.805	atmA

**Constants Used:**

Gas Constant, R: 0.08206 L\*atm/g-mol\*°K

**Calculated Parameters:**

Reduced Temperature,  $T_r$  0.995 ( $T/T_c$ )  
 Reduced Pressure,  $P_r$  0.164 ( $P/P_c$ )

**Equations:**

$A^2 =$	0.07100
B =	0.01427
q =	-0.05653
r =	0.00101
f =	-0.27681
g =	-0.05624
C =	0.00001
D =	0.31222
E1 =	0.02581
$\phi =$	IRRELEVANT
E =	0.29553
z1 =	IRRELEVANT
z2 =	IRRELEVANT
z3 =	IRRELEVANT
z0 =	0.94108
z =	<b>0.941</b>

Revision: 0

Prepared By:

Checked By:

Approved By:

## SI Units

### Inputs

Head Coefficient, $\mu$	0.48	dimensionless	(Recommended Value)
Calc. Polytropic Head per Stage, $H_{poly}$	23,990	N-m/kg	(Calculated value from 'Polytropic Head SI' worksheet)
Impeller diameter	762	mm	(Refer Table 1 on 'Polytropic Head SI' worksheet)

### Calculations

Impeller tip Speed, $u_2$	223.6	m/s
Approx. Rotational Speed, N	5,603	rpm

## US Customary Units

### Inputs

Head Coefficient, $\mu$	0.48	dimensionless	(Recommended Value)
Calc. Polytropic Head per Stage, $H_{poly}$	8,030	ft-lb <sub>f</sub> / lb <sub>m</sub>	(Calculated value from 'Polytropic Head US Cust' worksheet)
Impeller diameter	30	inch	(Refer Table 1 on 'Polytropic Head US Cust' worksheet)

### Calculations

Impeller tip Speed, $u_2$	733.9	fps
Approx. Rotational Speed, N	5,607	rpm