CHAPTER

7

Natural Convection Systems

7-1 | INTRODUCTION

Our previous discussions of convection heat transfer have considered only the calculation of forced-convection systems where the fluid is forced by or through the heat-transfer surface. Natural, or free, convection is observed as a result of the motion of the fluid due to density changes arising from the heating process. A hot radiator used for heating a room is one example of a practical device that transfers heat by free convection. The movement of the fluid in free convection, whether it is a gas or a liquid, results from the buoyancy forces imposed on the fluid when its density in the proximity of the heat-transfer surface is decreased as a result of the heating process. The buoyancy forces would not be present if the fluid were not acted upon by some external force field such as gravity, although gravity is not the only type of force field that can produce the free-convection currents; a fluid enclosed in a rotating machine is acted upon by a centrifugal force field, and thus could experience free-convection currents if one or more of the surfaces in contact with the fluid were heated. The buoyancy forces that give rise to the free-convection currents are called *body forces.*

7-2 | FREE-CONVECTION HEAT TRANSFER ON A VERTICAL FLAT PLATE

Consider the vertical flat plate shown in Figure 7-1. When the plate is heated, a free-convection boundary layer is formed, as shown. The velocity profile in this boundary layer is quite unlike the velocity profile in a forced-convection boundary layer. At the wall the velocity is zero because of the no-slip condition; it increases to some maximum value and then decreases to zero at the edge of the boundary layer since the "free-stream" conditions are at rest in the free-convection system. The initial boundary-layer development is laminar; but at some distance from the leading edge, depending on the fluid properties and the temperature difference between wall and environment, turbulent eddies are formed, and transition to a turbulent boundary layer begins. Farther up the plate the boundary layer may become fully turbulent.

To analyze the heat-transfer problem, we must first obtain the differential equation of motion for the boundary layer. For this purpose we choose the x coordinate along the plate and the y coordinate perpendicular to the plate as in the analyses of Chapter 5. The only new force that must be considered in the derivation is the weight of the element of fluid.

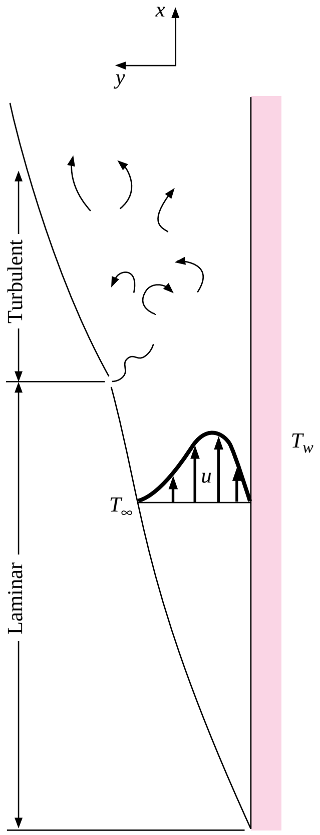


Figure 7-1 | Boundary layer on a vertical flat plate.

As before, we equate the sum of the external forces in the x direction to the change in momentum flux through the control volume $dx\,dy$. There results

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}-\rho g+\mu\frac{\partial^2 u}{\partial y^2} \qquad [7\text{-}1]$$

where the term $-\rho g$ represents the weight force exerted on the element. The pressure gradient in the x direction results from the change in elevation up the plate. Thus

$$\frac{\partial p}{\partial x}=-\rho_\infty g \qquad [7\text{-}2]$$

In other words, the change in pressure over a height dx is equal to the weight per unit area of the fluid element. Substituting Equation (7-2) into Equation (7-1) gives

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=g(\rho_\infty-\rho)+\mu\frac{\partial^2 u}{\partial y^2} \qquad [7\text{-}3]$$

The density difference $\rho_\infty-\rho$ may be expressed in terms of the volume coefficient of expansion β, defined by

$$\beta=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p=\frac{1}{V_\infty}\frac{V-V_\infty}{T-T_\infty}=\frac{\rho_\infty-\rho}{\rho(T-T_\infty)}$$

so that

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=g\rho\beta(T-T_\infty)+\mu\frac{\partial^2 u}{\partial y^2} \qquad [7\text{-}4]$$

This is the equation of motion for the free-convection boundary layer. Notice that the solution for the velocity profile demands a knowledge of the temperature distribution. The energy equation for the free-convection system is the same as that for a forced-convection system at low velocity:

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2} \qquad [7\text{-}5]$$

The volume coefficient of expansion β may be determined from tables of properties for the specific fluid. For ideal gases it may be calculated from (see Problem 7-3)

$$\beta=\frac{1}{T}$$

where T is the absolute temperature of the gas.

Even though the fluid motion is the result of density variations, these variations are quite small, and a satisfactory solution to the problem may be obtained by assuming incompressible flow, that is, $\rho=\text{constant}$. To effect a solution of the equation of motion, we use the integral method of analysis similar to that used in the forced-convection problem of Chapter 5. Detailed boundary-layer analyses have been presented in References 13, 27, and 32.

For the free-convection system, the integral momentum equation becomes

$$\frac{d}{dx}\left(\int_0^\delta \rho u^2\,dy\right)=-\tau_w+\int_0^\delta \rho g\beta(T-T_\infty)\,dy$$
$$=-\mu\left.\frac{\partial u}{\partial y}\right]_{y=0}+\int_0^\delta \rho g\beta(T-T_\infty)\,dy \qquad [7\text{-}6]$$

and we observe that the functional form of both the velocity and the temperature distributions must be known in order to arrive at the solution. To obtain these functions, we proceed in much the same way as in Chapter 5. The following conditions apply for the temperature distribution:

$$T = T_w \quad \text{at } y = 0$$
$$T = T_\infty \quad \text{at } y = \delta$$
$$\frac{\partial T}{\partial y} = 0 \quad \text{at } y = \delta$$

so that we obtain for the temperature distribution

$$\frac{T - T_\infty}{T_w - T_\infty} = \left(1 - \frac{y}{\delta}\right)^2 \qquad \textbf{[7-7]}$$

Three conditions for the velocity profile are

$$u = 0 \quad \text{at } y = 0$$
$$u = 0 \quad \text{at } y = \delta$$
$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = \delta$$

An additional condition may be obtained from Equation (7-4) by noting that

$$\frac{\partial^2 u}{\partial y^2} = -g\beta \frac{T_w - T_\infty}{\nu} \qquad \text{at } y = 0$$

As in the integral analysis for forced-convection problems, we assume that the velocity profiles have geometrically similar shapes at various x distances along the plate. For the free-convection problem, we now assume that the velocity may be represented as a polynomial function of y multiplied by some arbitrary function of x. Thus,

$$\frac{u}{u_x} = a + by + cy^2 + dy^3$$

where u_x is a fictitious velocity that is a function of x. The cubic-polynomial form is chosen because there are four conditions to satisfy, and this is the simplest type of function that may be used. Applying the four conditions to the velocity profile listed above, we have

$$\frac{u}{u_x} = \frac{\beta\delta^2 g(T_w - T_\infty)}{4u_x\nu} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2$$

The term involving the temperature difference, δ^2, and u_x may be incorporated into the function u_x so that the final relation to be assumed for the velocity profile is

$$\frac{u}{u_x} = \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2 \qquad \textbf{[7-8]}$$

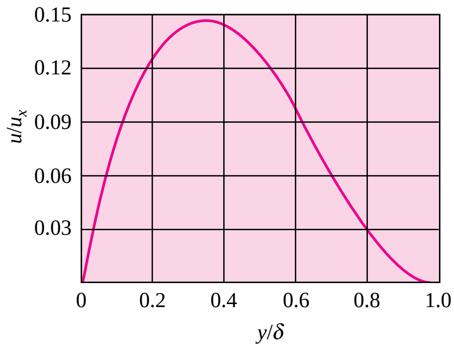
A plot of Equation (7-8) is given in Figure 7-2. Substituting Equations (7-7) and (7-8) into Equation (7-6) and carrying out the integrations and differentiations yields

$$\frac{1}{105}\frac{d}{dx}(u_x^2\delta) = \frac{1}{3}g\beta(T_w - T_\infty)\delta - \nu\frac{u_x}{\delta} \qquad \textbf{[7-9]}$$

The integral form of the energy equation for the free-convection system is

$$\frac{d}{dx}\left[\int_0^\delta u(T - T_\infty)dy\right] = -\alpha \left.\frac{dT}{dy}\right]_{y=0} \qquad \textbf{[7-10]}$$

Figure 7-2 | Free-convection velocity profile given by Equation (7-8).



and when the assumed velocity and temperature distributions are inserted into this equation and the operations are performed, there results

$$\frac{1}{30}(T_w - T_\infty)\frac{d}{dx}(u_x\delta) = 2\alpha\frac{T_w - T_\infty}{\delta} \quad \textbf{[7-11]}$$

It is clear from the reasoning that led to Equation (7-8) that

$$u_x \sim \delta^2 \quad \textbf{[7-12]}$$

Inserting this type of relation in Equation (7-9) yields the result that

$$\delta \sim x^{1/4} \quad \textbf{[7-13]}$$

We therefore assume the following exponential functional variations for u_x and δ:

$$u_x = C_1 x^{1/2} \quad \textbf{[7-14]}$$

$$\delta = C_2 x^{1/4} \quad \textbf{[7-15]}$$

Introducing these relations into Equations (7-9) and (7-11) gives

$$\frac{5}{420}C_1^2 C_2 x^{1/4} = g\beta(T_w - T_\infty)\frac{C_2}{3}x^{1/4} - \frac{C_1}{C_2}\nu x^{1/4} \quad \textbf{[7-16]}$$

and

$$\frac{1}{40}C_1 C_2 x^{-1/4} = \frac{2\alpha}{C_2}x^{-1/4} \quad \textbf{[7-17]}$$

These two equations may be solved for the constants C_1 and C_2 to give

$$C_1 = 5.17\nu\left(\frac{20}{21} + \frac{\nu}{\alpha}\right)^{-1/2}\left[\frac{g\beta(T_w - T_\infty)}{\nu^2}\right]^{1/2} \quad \textbf{[7-18]}$$

$$C_2 = 3.93\left(\frac{20}{21} + \frac{\nu}{\alpha}\right)^{1/4}\left[\frac{g\beta(T_w - T_\infty)}{\nu^2}\right]^{-1/4}\left(\frac{\nu}{\alpha}\right)^{-1/2} \quad \textbf{[7-19]}$$

The resultant expressions for the boundary layer thickness and fictitious velocity u_x are

$$\frac{\delta}{x} = 3.93\,\mathrm{Pr}^{-1/2}(0.952 + \mathrm{Pr})^{1/4}\mathrm{Gr}_x^{-1/4} \quad \textbf{[7-20a]}$$

$$u_x \frac{x}{\nu} = 5.17(0.952 + \text{Pr})^{-1/2}\text{Gr}_x^{1/2} \qquad \textbf{[7-20b]}$$

The velocity profile shown in Figure 7-2 has its maximum value at $y/\delta = 1/3$, giving $u_{\text{max}} = (4/27)u_x = 0.148u_x$. The mass flow through the boundary layer at any x position may be determined by evaluating the integral

$$\dot{m} = \int \rho u dy = \int_0^\delta \rho u_x \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2 dy = \frac{1}{12}\rho u_x \delta = 0.083\rho u_x \delta = \frac{9}{16}\rho u_{\text{max}}\delta \qquad \textbf{[7-20c]}$$

The respective values of δ and u_x determined from Equations (7-20*a*) and (7-20*b*) may be inserted to obtain the mass flow values.

The Prandtl number $\text{Pr} = \nu/\alpha$ has been introduced in the above expressions along with a new dimensionless group called the *Grashof number* Gr_x:

$$\text{Gr}_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2} \qquad \textbf{[7-21]}$$

The heat-transfer coefficient may be evaluated from

$$q_w = -kA\left.\frac{dT}{dy}\right]_w = hA(T_w - T_\infty)$$

Using the temperature distribution of Equation (7-7), one obtains

$$h = \frac{2k}{\delta} \qquad \text{or} \qquad \frac{hx}{k} = \text{Nu}_x = 2\frac{x}{\delta}$$

so that the dimensionless equation for the heat-transfer coefficient becomes

$$\text{Nu}_x = 0.508\,\text{Pr}^{1/2}(0.952 + \text{Pr})^{-1/4}\text{Gr}_x^{1/4} \qquad \textbf{[7-22]}$$

Equation (7-22) gives the variation of the local heat-transfer coefficient along the vertical plate. The average heat-transfer coefficient may then be obtained by performing the integration

$$\bar{h} = \frac{1}{L}\int_0^L h_x dx \qquad \textbf{[7-23]}$$

For the variation given by Equation (7-22), the average coefficient is

$$\bar{h} = \tfrac{4}{3}h_{x=L} \qquad \textbf{[7-24]}$$

The Grashof number may be interpreted physically as a dimensionless group representing the ratio of the buoyancy forces to the viscous forces in the free-convection flow system. It has a role similar to that played by the Reynolds number in forced-convection systems and is the primary variable used as a criterion for transition from laminar to turbulent boundary-layer flow. For air in free convection on a vertical flat plate, the critical Grashof number has been observed by Eckert and Soehngen [1] to be approximately 4×10^8. Values ranging between 10^8 and 10^9 may be observed for different fluids and environment "turbulence levels."

A very complete survey of the stability and transition of free-convection boundary layers has been given by Gebhart et al. [13–15].

The foregoing analysis of free-convection heat transfer on a vertical flat plate is the simplest case that may be treated mathematically, and it has served to introduce the new

Figure 7-3 | Pulsed free-convection boundary layer on vertical flat plate. Distance between letters = 5 cm.

dimensionless variable, the Grashof number,† which is important in all free-convection problems. But as in some forced-convection problems, experimental measurements must be relied upon to obtain relations for heat transfer in other circumstances. These circumstances are usually those in which it is difficult to predict temperature and velocity profiles analytically. Turbulent free convection is an important example, just as is turbulent forced convection, of a problem area in which experimental data are necessary; however, the problem is more acute with free-convection flow systems than with forced-convection systems because the velocities are usually so small that they are very difficult to measure. For example, the maximum free-convection velocity experienced by a vertical plate heated to 45°C and exposed to atmospheric room air at 25°C is only about 350 mm/s. Despite the experimental difficulties, velocity measurements have been performed using hydrogen-bubble techniques [26], hot-wire anemometry [28], and quartz-fiber anemometers. Temperature field measurements have been obtained through the use of the Zehnder-Mach interferometer. The laser anemometer [29] is particularly useful for free-convection measurements because it does not disturb the flow field.

An interferometer indicates lines of constant density in a fluid flow field. For a gas in free convection at low pressure these lines of constant density are equivalent to lines of constant temperature. Once the temperature field is obtained, the heat transfer from a surface in free convection may be calculated by using the temperature gradient at the surface and the thermal conductivity of the gas. Several interferometric studies of free convection have been made [1–3], and Figure 7-3 indicates the isotherms in a free-convection boundary layer on a vertical flat plate with $T_W = 48$°C and $T_\infty = 20$°C in room air. The spacing between the horizontal markers is about 2.5 cm, indicating a boundary-layer thickness of about that same value. The letter A corresponds to the leading edge of the plate. Note that the isotherms are more closely spaced near the plate surface, indicating a higher temperature gradient in that region. The oscillatory or "wave" shape of the boundary layer isotherms is caused by a heat pulse from a fine wire located at $x = 2.5$ cm and having a frequency of about 2.5 Hz. The pulse moves up the plate at about the boundary layer velocity, so an indication of the velocity profile may be obtained by connecting the maximum points in the isotherms. Such a profile is indicated in Figure 7-4. Eventually, at about $\text{Gr} = 10^8$–10^9 small oscillations in the boundary layer become amplified and transition to turbulence begins. The region shown in Figure 7-3 is all laminar.

A number of references treat the various theoretical and empirical aspects of free-convection problems. One of the most extensive discussions is given by Gebhart et. al. [13], and the interested reader may wish to consult this reference for additional information.

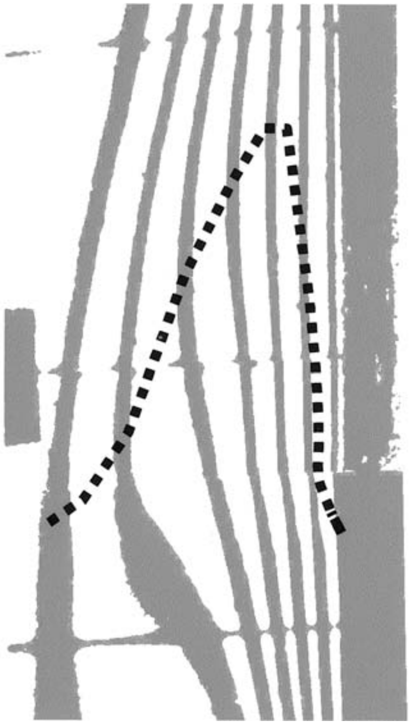
7-3 | EMPIRICAL RELATIONS FOR FREE CONVECTION

Over the years it has been found that average free-convection heat-transfer coefficients can be represented in the following functional form for a variety of circumstances:

$$\overline{\text{Nu}}_f = C(\text{Gr}_f \, \text{Pr}_f)^m \qquad \textbf{[7-25]}$$

† History is not clear on the point, but it appears that the Grashof number was named for Franz Grashof, a professor of applied mechanics at Karlsruhe around 1863 and one of the founding directors of *Verein deutscher Ingenieure* in 1855. He developed some early steam-flow formulas but made no significant contributions to free convection [36].

Figure 7-4 | Free-convection velocity profile indicated by connecting maximum points in boundary-layer isotherms of Figure 7-3.



where the subscript f indicates that the properties in the dimensionless groups are evaluated at the film temperature

$$T_f = \frac{T_\infty + T_w}{2}$$

The product of the Grashof and Prandtl numbers is called the Rayleigh number:

$$\text{Ra} = \text{Gr Pr} \qquad \textbf{[7-26]}$$

Characteristic Dimensions

The characteristic dimension to be used in the Nusselt and Grashof numbers depends on the geometry of the problem. For a vertical plate it is the height of the plate L; for a horizontal cylinder it is the diameter d; and so forth. Experimental data for free-convection problems appear in a number of references, with some conflicting results. The purpose of the sections that follow is to give these results in a summary form that may be easily used for calculation purposes. The functional form of Equation (7-25) is used for many of these presentations, with the values of the constants C and m specified for each case. Table 7-1 provides a summary of the values of these correlation constants for different geometries, and the sections that follow discuss the correlations in more detail.

For convenience of the reader, the present author has presented a graphical meld of the correlations for the isothermal vertical plate and horizontal cylinder configurations in the form of Figures 7-5 and 7-6. These figures may be used in lieu of the formulas when a quick estimate of performance is desired.

Table 7-1 | Constants for use with Equation (7-25) for isothermal surfaces.

Geometry	$Gr_f\ Pr_f$	C	m	Reference(s)
Vertical planes and cylinders	10^{-1}–10^{4}	Use Fig. 7-5	Use Fig. 7-5	4
	10^{4}–10^{9}	0.59	$\frac{1}{4}$	4
	10^{9}–10^{13}	0.021	$\frac{2}{5}$	30
	10^{9}–10^{13}	0.10	$\frac{1}{3}$	22, 16†
Horizontal cylinders	0–10^{-5}	0.4	0	4
	10^{-5}–10^{4}	Use Fig. 7-6	Use Fig. 7-6	4
	10^{4}–10^{9}	0.53	$\frac{1}{4}$	4
	10^{9}–10^{12}	0.13	$\frac{1}{3}$	4
	10^{-10}–10^{-2}	0.675	0.058	76†
	10^{-2}–10^{2}	1.02	0.148	76†
	10^{2}–10^{4}	0.850	0.188	76
	10^{4}–10^{7}	0.480	$\frac{1}{4}$	76
	10^{7}–10^{12}	0.125	$\frac{1}{3}$	76
Upper surface of heated plates or lower surface of cooled plates	2×10^{4}–8×10^{6}	0.54	$\frac{1}{4}$	44, 52
Upper surface of heated plates or lower surface of cooled plates	8×10^{6}–10^{11}	0.15	$\frac{1}{3}$	44, 52
Lower surface of heated plates or upper surface of cooled plates	10^{5}–10^{11}	0.27	$\frac{1}{4}$	44, 37, 75
Vertical cylinder, height = diameter characteristic length = diameter	10^{4}–10^{6}	0.775	0.21	77
Irregular solids, characteristic length = distance fluid particle travels in boundary layer	10^{4}–10^{9}	0.52	$\frac{1}{4}$	78

† Preferred.

7-4 | FREE CONVECTION FROM VERTICAL PLANES AND CYLINDERS

Isothermal Surfaces

For vertical surfaces, the Nusselt and Grashof numbers are formed with L, the height of the surface as the characteristic dimension. If the boundary-layer thickness is not large compared with the diameter of the cylinder, the heat transfer may be calculated with the same relations used for vertical plates. The general criterion is that a vertical cylinder may be treated as a vertical flat plate [13] when

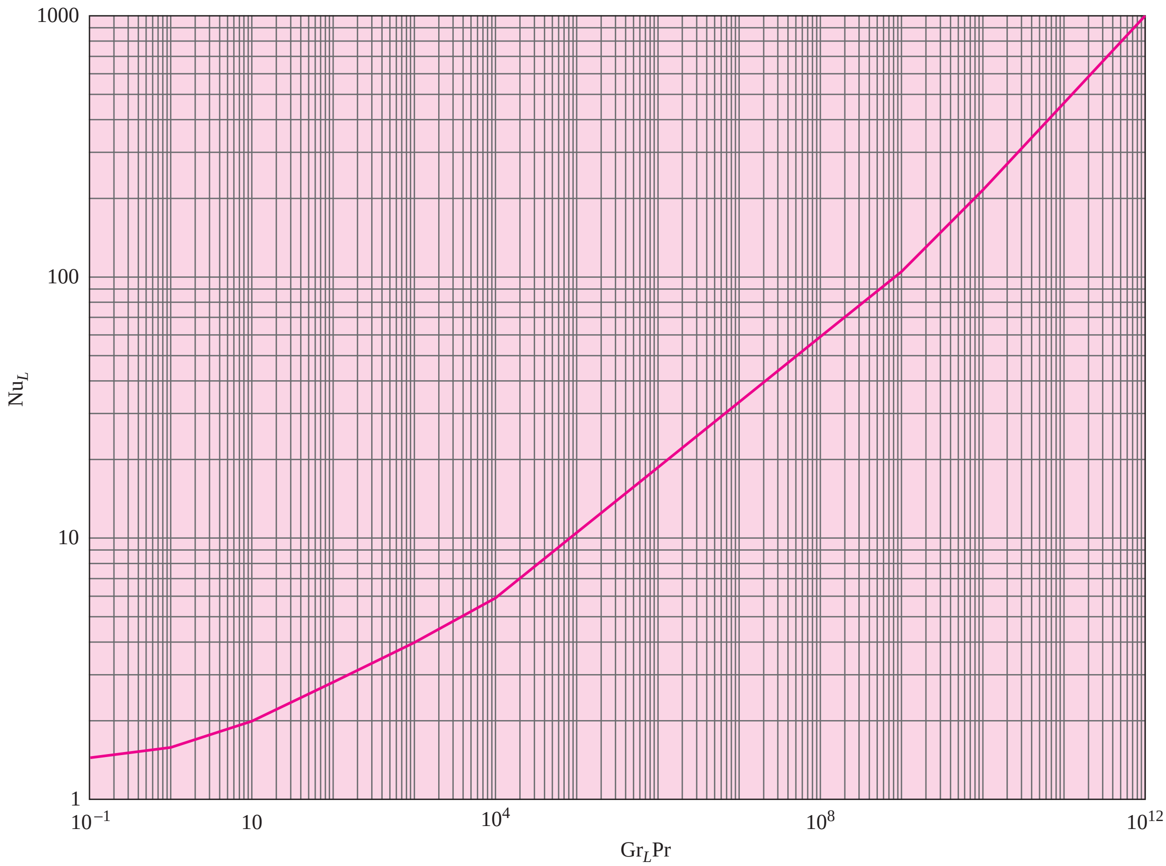
$$\frac{D}{L} \geq \frac{35}{Gr_L^{1/4}} \qquad \textbf{[7-27]}$$

where D is the diameter of the cylinder. For vertical cylinders too small to meet this criteria, the analysis of Reference [84] for gases with $Pr = 0.7$ indicates that the flat plate results for the average heat-transfer coefficient should be multiplied by a factor F to account for the curvature, where

$$F = 1.3[(L/D)/Gr_D]^{1/4} + 1.0 \qquad \textbf{[7-27}\textbf{\textit{a}}\textbf{]}$$

For *isothermal* surfaces, the values of the constants C and m are given in Table 7-1 with the appropriate references noted for further consultation. The reader's attention is directed

Figure 7-5 | Free-convection heat transfer from vertical isothermal plates.



to the two sets of constants given for the turbulent case ($Gr_f\, Pr_f > 10^9$). Although there may appear to be a decided difference in these constants, a comparison by Warner and Arpaci [22] of the two relations with experimental data indicates that both sets of constants fit available data. There are some indications from the analytical work of Bayley [16], as well as heat flux measurements of Reference 22, that the relation

$$Nu_f = 0.10(Gr_f\, Pr_f)^{1/3}$$

may be preferable.

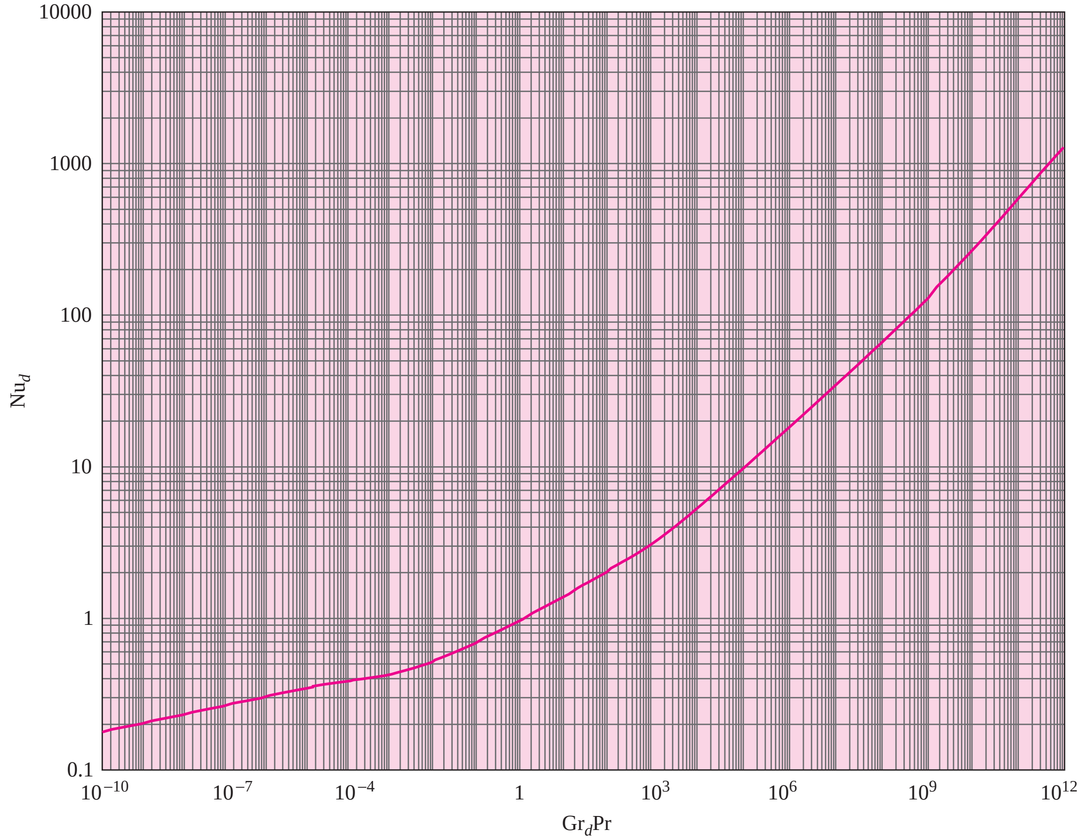
More complicated relations have been provided by Churchill and Chu [71] that are applicable over wider ranges of the Rayleigh number:

$$\overline{Nu} = 0.68 + \frac{0.670\ Ra^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \qquad \text{for } Ra_L < 10^9 \qquad \textbf{[7-28]}$$

$$\overline{Nu}^{1/2} = 0.825 + \frac{0.387\ Ra^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \qquad \text{for } 10^{-1} < Ra_L < 10^{12} \qquad \textbf{[7-29]}$$

Equation (7-28) is also a satisfactory representation for constant heat flux. Properties for these equations are evaluated at the film temperature.

Figure 7-6 | Free-convection heat transfer from horizontal isothermal cylinders.



Constant-Heat-Flux Surfaces

Extensive experiments have been reported in References 25, 26, and 39 for free convection from vertical and inclined surfaces to water under constant-heat-flux conditions. In such experiments, the results are presented in terms of a modified Grashof number, Gr*:

$$\mathrm{Gr}_x^* = \mathrm{Gr}_x\ \mathrm{Nu}_x = \frac{g\beta q_w x^4}{k\nu^2} \qquad \textbf{[7-30]}$$

where $q_w = q/A$ is the heat flux per unit area and is assumed constant over the entire plate surface area.

The *local* heat-transfer coefficients were correlated by the following relation for the laminar range:

$$\mathrm{Nu}_{xf} = \frac{hx}{k_f} = 0.60(\mathrm{Gr}_x^*\ \mathrm{Pr}_f)^{1/5} \qquad 10^5 < \mathrm{Gr}_x^*\ \mathrm{Pr} < 10^{11};\ q_w = \mathrm{const} \qquad \textbf{[7-31]}$$

It is to be noted that the criterion for laminar flow expressed in terms of Gr_x^* is not the same as that expressed in terms of Gr_x. Boundary-layer transition was observed to begin between $\mathrm{Gr}_x^*\ \mathrm{Pr} = 3 \times 10^{12}$ and 4×10^{13} and to end between 2×10^{13} and 10^{14}. Fully developed turbulent flow was present by $\mathrm{Gr}_x^*\ \mathrm{Pr} = 10^{14}$, and the experiments were extended up to $\mathrm{Gr}_x^*\ \mathrm{Pr} = 10^{16}$. For the turbulent region, the local heat-transfer coefficients are correlated with

$$\mathrm{Nu}_x = 0.17(\mathrm{Gr}_x^*\ \mathrm{Pr})^{1/4} \qquad 2 \times 10^{13} < \mathrm{Gr}_x^*\ \mathrm{Pr} < 10^{16};\ q_w = \mathrm{const} \qquad \textbf{[7-32]}$$

All properties in Equations (7-31) and (7-32) are evaluated at the local film temperature. Although these experiments were conducted for water, the resulting correlations are shown to work for air as well. The average heat-transfer coefficient for the constant-heat-flux case may not be evaluated from Equation (7-24) but must be obtained through a separate application of Equation (7-23). Thus, for the laminar region, using Equation (7-31) to evaluate h_x,

$$\bar{h} = \frac{1}{L}\int_0^L h_x dx$$

$$\bar{h} = \tfrac{5}{4}h_{x=L} \qquad q_w = \text{const}$$

At this point we may note the relationship between the correlations in the form of Equation (7-25) and those just presented in terms of $\text{Gr}_x^* = \text{Gr}_x \, \text{Nu}_x$. Writing Equation (7-25) as a *local* heat-transfer form gives

$$\text{Nu}_x = C(\text{Gr}_x \, \text{Pr})^m \qquad \textbf{[7-33]}$$

Inserting $\text{Gr}_x = \text{Gr}_x^*/\text{Nu}_x$ gives

$$\text{Nu}_x^{1+m} = C(\text{Gr}_x^* \, \text{Pr})^m$$

or

$$\text{Nu}_x = C^{1/(1+m)}(\text{Gr}_x^* \, \text{Pr})^{m/(1+m)} \qquad \textbf{[7-34]}$$

Thus, when the "characteristic" values of m for laminar and turbulent flow are compared to the exponents on Gr_x^*, we obtain

Laminar, $m = \dfrac{1}{4}$: $\qquad \dfrac{m}{1+m} = \dfrac{1}{5}$

Turbulent, $m = \dfrac{1}{3}$: $\qquad \dfrac{m}{1+m} = \dfrac{1}{4}$

While the Gr^* formulation is easier to employ for the constant-heat-flux case, we see that the characteristic exponents fit nicely into the scheme that is presented for the isothermal surface correlations.

It is also interesting to note the variation of h_x with x in the two characteristic regimes. For the laminar range $m = \frac{1}{4}$, and from Equation (7-25)

$$h_x \sim \frac{1}{x}(x^3)^{1/4} = x^{-1/4}$$

In the turbulent regime $m = \frac{1}{3}$, and we obtain

$$h_x \sim \frac{1}{x}(x^3)^{1/3} = \text{const with } x$$

So when turbulent free convection is encountered, the local heat-transfer coefficient is essentially constant with x.

Churchill and Chu [71] show that Equation (7-28) may be modified to apply to the constant-heat-flux case if the average Nusselt number is based on the wall heat flux and the temperature difference at the center of the plate $(x = L/2)$. The result is

$$\overline{\text{Nu}}_L^{1/4}(\overline{\text{Nu}}_L - 0.68) = \frac{0.67(\text{Gr}_L^* \, \text{Pr})^{1/4}}{[1 + (0.492/\text{Pr})^{9/16}]^{4/9}} \qquad \textbf{[7-35]}$$

where $\overline{\text{Nu}}_L = q_w L/(k\overline{\Delta T})$ and $\overline{\Delta T} = T_w - T_\infty$ at $L/2 - T_\infty$.

EXAMPLE 7-1 Constant Heat Flux from Vertical Plate

In a plant location near a furnace, a net radiant energy flux of 800 W/m^2 is incident on a vertical metal surface 3.5 m high and 2 m wide. The metal is insulated on the back side and painted black so that all the incoming radiation is lost by free convection to the surrounding air at 30°C. What average temperature will be attained by the plate?

■ Solution

We treat this problem as one with constant heat flux on the surface. Since we do not know the surface temperature, we must make an estimate for determining T_f and the air properties. An *approximate* value of h for free-convection problems is 10 W/m^2 · °C, and so, *approximately,*

$$\Delta T = \frac{q_w}{h} \approx \frac{800}{10} = 80°\text{C}$$

Then

$$T_f \approx \frac{80}{2} + 30 = 70°\text{C} = 343\ \text{K}$$

At 70°C the properties of air are

$$\nu = 2.043 \times 10^{-5}\ \text{m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = 2.92 \times 10^{-3}\ \text{K}^{-1}$$

$$k = 0.0295\ \text{W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7$$

From Equation (7-30), with $x = 3.5$ m,

$$\text{Gr}_x^* = \frac{g\beta q_w x^4}{k\nu^2} = \frac{(9.8)(2.92 \times 10^{-3})(800)(3.5)^4}{(0.0295)(2.043 \times 10^{-5})^2} = 2.79 \times 10^{14}$$

We may therefore use Equation (7-32) to evaluate h_x:

$$\begin{aligned} h_x &= \frac{k}{x}(0.17)(\text{Gr}_x^*\,\text{Pr})^{1/4} \\ &= \frac{0.0295}{3.5}(0.17)(2.79 \times 10^{14} \times 0.7)^{1/4} \\ &= 5.36\ \text{W/m}^2 \cdot °\text{C}\ [0.944\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}] \end{aligned}$$

In the turbulent heat transfer governed by Equation (7-32), we note that

$$\text{Nu}_x = \frac{hx}{k} \sim (\text{Gr}_x^*)^{1/4} \sim (x^4)^{1/4}$$

or h_x does not vary with x, and we may take this as the average value. The value of $h = 5.41$ W/m^2 · °C is less than the approximate value we used to estimate T_f. Recalculating ΔT, we obtain

$$\Delta T = \frac{q_w}{h} = \frac{800}{5.36} = 149°\text{C}$$

Our new film temperature would be

$$T_f = 30 + \frac{149}{2} = 104.5°\text{C}$$

At 104.5°C the properties of air are

$$\nu = 2.354 \times 10^{-5}\ \text{m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = 2.65 \times 10^{-3}/\text{K}$$

$$k = 0.0320\ \text{W/m} \cdot °\text{C} \qquad \text{Pr} = 0.695$$

Then

$$\mathrm{Gr}_x^* = \frac{(9.8)(2.65 \times 10^{-3})(800)(3.5)^4}{(0.0320)(2.354 \times 10^{-5})^2} = 1.75 \times 10^{14}$$

and h_x is calculated from

$$\begin{aligned} h_x &= \frac{k}{x}(0.17)(\mathrm{Gr}_x^*\,\mathrm{Pr})^{1/4} \\ &= \frac{(0.0320)(0.17)}{3.5}[(1.758 \times 10^{14})(0.695)]^{1/4} \\ &= 5.17\ \mathrm{W/m^2 \cdot {}^\circ C}\ [-0.91\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}] \end{aligned}$$

Our new temperature difference is calculated as

$$\Delta T = (T_w - T_\infty)_{\mathrm{av}} = \frac{q_w}{h} = \frac{800}{5.17} = 155^\circ\mathrm{C}$$

The average wall temperature is therefore

$$T_{w,\mathrm{av}} = 155 + 30 = 185^\circ\mathrm{C}$$

Another iteration on the value of T_f is not warranted by the improved accuracy that would result.

Heat Transfer from Isothermal Vertical Plate

EXAMPLE 7-2

A large vertical plate 4.0 m high is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer if the plate is 10 m wide.

Solution

We first determine the film temperature as

$$T_f = \frac{60 + 10}{2} = 35^\circ\mathrm{C} = 308\ \mathrm{K}$$

The properties of interest are thus

$$\beta = \frac{1}{308} = 3.25 \times 10^{-3} \qquad k = 0.02685$$

$$\nu = 16.5 \times 10^{-6} \qquad \mathrm{Pr} = 0.7$$

and

$$\begin{aligned} \mathrm{Gr\,Pr} &= \frac{(9.8)(3.25 \times 10^{-3})(60 - 10)(4)^3}{(16.5 \times 10^{-6})^2} 0.7 \\ &= 2.62 \times 10^{11} \end{aligned}$$

We then may use Equation (7-29) to obtain

$$\begin{aligned} \overline{\mathrm{Nu}}^{1/2} &= 0.825 + \frac{(0.387)(2.62 \times 10^{11})^{1/6}}{[1 + (0.492/0.7)^{9/16}]^{8/27}} \\ &= 26.75 \\ \overline{\mathrm{Nu}} &= 716 \end{aligned}$$

The heat-transfer coefficient is then

$$\bar{h} = \frac{(716)(0.02685)}{4.0} = 4.80\ \mathrm{W/m^2 \cdot {}^\circ C}$$

The heat transfer is

$$q = \bar{h}A(T_w - T_\infty)$$
$$= (4.80)(4)(10)(60 - 10) = 9606 \text{ W}$$

As an alternative, we could employ the simpler relation

$$\text{Nu} = 0.10(\text{Gr Pr})^{1/3}$$
$$= (0.10)(2.62 \times 10^{11})^{1/3} = 639.9$$

which gives a value about 10 percent lower than Equation (7-29).

7-5 | FREE CONVECTION FROM HORIZONTAL CYLINDERS

The values of the constants C and m are given in Table 7-1 according to References 4 and 76. The predictions of Morgan (Reference 76 in Table 7-1) are the most reliable for Gr Pr of approximately 10^{-5}. A more complicated expression for use over a wider range of Gr Pr is given by Churchill and Chu [70]:

$$\overline{\text{Nu}}^{1/2} = 0.60 + 0.387\left\{\frac{\text{Gr Pr}}{[1 + (0.559/\text{Pr})^{9/16}]^{16/9}}\right\}^{1/6} \quad \text{for } 10^{-5} < \text{Gr Pr} < 10^{12} \qquad \textbf{[7-36]}$$

A simpler equation is available from Reference 70 but is restricted to the laminar range of $10^{-6} < \text{Gr Pr} < 10^9$:

$$\text{Nu}_d = 0.36 + \frac{0.518(\text{Gr}_d\,\text{Pr})^{1/4}}{[1 + (0.559/\text{Pr})^{9/16}]^{4/9}} \qquad \textbf{[7-37]}$$

Properties in Equations (7-36) and (7-37) are evaluated at the film temperature.

Heat transfer from horizontal cylinders to liquid metals may be calculated from Reference 46:

$$\text{Nu}_d = 0.53(\text{Gr}_d\,\text{Pr}^2)^{1/4} \qquad \textbf{[7-38]}$$

EXAMPLE 7-3 Heat Transfer from Horizontal Tube in Water

A 2.0-cm-diameter horizontal heater is maintained at a surface temperature of 38°C and submerged in water at 27°C. Calculate the free-convection heat loss per unit length of the heater.

■ Solution

The film temperature is

$$T_f = \frac{38 + 27}{2} = 32.5°\text{C}$$

From Appendix A the properties of water are

$$k = 0.630 \text{ W/m} \cdot °\text{C}$$

and the following term is particularly useful in obtaining the Gr Pr product when it is multiplied by $d^3\Delta T$:

$$\frac{g\beta\rho^2 c_p}{\mu k} = 2.48 \times 10^{10} \qquad [1/\text{m}^3 \cdot °\text{C}]$$

$$\text{Gr Pr} = (2.48 \times 10^{10})(38 - 27)(0.02)^3 = 2.18 \times 10^6$$

Using Table 7-1, we get $C = 0.53$ and $m = \frac{1}{4}$, so that

$$\text{Nu} = (0.53)(2.18 \times 10^6)^{1/4} = 20.36$$

$$h = \frac{(20.36)(0.63)}{0.02} = 642\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

The heat transfer is thus

$$\frac{q}{L} = h\pi d(T_w - T_\infty)$$
$$= (642)\pi(0.02)(38 - 27) = 443\ \text{W/m}$$

Heat Transfer from Fine Wire in Air

EXAMPLE 7-4

A fine wire having a diameter of 0.02 mm is maintained at a constant temperature of 54°C by an electric current. The wire is exposed to air at 1 atm and 0°C. Calculate the electric power necessary to maintain the wire temperature if the length is 50 cm.

■ Solution

The film temperature is $T_f = (54 + 0)/2 = 27°\text{C} = 300$ K, so the properties are

$$\beta = 1/300 = 0.00333 \qquad \nu = 15.69 \times 10^{-6}\ \text{m}^2/\text{s}$$
$$k = 0.02624\ \text{W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 0.708$$

The Gr Pr product is then calculated as

$$\text{Gr Pr} = \frac{(9.8)(0.00333)(54 - 0)(0.02 \times 10^{-3})^3}{(15.69 \times 10^{-6})^2}(0.708) = 4.05 \times 10^{-5}$$

From Table 7-1 we find $C = 0.675$ and $m = 0.058$ so that

$$\overline{\text{Nu}} = (0.675)(4.05 \times 10^{-5})^{0.058} = 0.375$$

and

$$\bar{h} = \overline{\text{Nu}}\left(\frac{k}{d}\right) = \frac{(0.375)(0.02624)}{0.02 \times 10^{-3}} = 492.6\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

The heat transfer or power required is then

$$q = \bar{h}A\,(T_w - T_\infty) = (492.6)\pi(0.02 \times 10^{-3})(0.5)(54 - 0) = 0.836\ \text{W}$$

Heated Horizontal Pipe in Air

EXAMPLE 7-5

A horizontal pipe 1 ft (0.3048 m) in diameter is maintained at a temperature of 250°C in a room where the ambient air is at 15°C. Calculate the free-convection heat loss per meter of length.

■ Solution

We first determine the Grashof-Prandtl number product and then select the appropriate constants from Table 7-1 for use with Equation (7-25). The properties of air are evaluated at the film temperature:

$$T_f = \frac{T_w + T_\infty}{2} = \frac{250 + 15}{2} = 132.5°\text{C} = 405.5\ \text{K}$$

$$k = 0.03406 \text{ W/m} \cdot {}^\circ\text{C} \qquad \beta = \frac{1}{T_f} = \frac{1}{405.5} = 2.47 \times 10^{-3} \text{ K}^{-1}$$

$$\nu = 26.54 \times 10^{-6} \text{ m}^2/\text{s} \qquad \text{Pr} = 0.687$$

$$\begin{aligned} \text{Gr}_d \, \text{Pr} &= \frac{g\beta(T_w - T_\infty)d^3}{\nu^2}\text{Pr} \\ &= \frac{(9.8)(2.47 \times 10^{-3})(250 - 15)(0.3048)^3(0.687)}{(26.54 \times 10^{-6})^2} \\ &= 1.571 \times 10^8 \end{aligned}$$

From Table 7-1, $C = 0.53$ and $m = \frac{1}{4}$, so that

$$\text{Nu}_d = 0.53(\text{Gr}_d \, \text{Pr})^{1/4} = (0.53)(1.571 \times 10^8)^{1/4} = 59.4$$

$$h = \frac{k\text{Nu}_d}{d} = \frac{(0.03406)(59.4)}{0.3048} = 6.63 \text{ W/m}^2 \cdot {}^\circ\text{C} \quad [1.175 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}]$$

The heat transfer per unit length is then calculated from

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = 6.63\pi(0.3048)(250 - 15) = 1.49 \text{ kW/m} \quad [1560 \text{ Btu/h} \cdot \text{ft}]$$

As an alternative, we could employ the more complicated expression, Equation (7-36), for solution of the problem. The Nusselt number thus would be calculated as

$$\overline{\text{Nu}}^{1/2} = 0.60 + 0.387 \left\{ \frac{1.571 \times 10^8}{[1 + (0.559/0.687)^{9/16}]^{16/9}} \right\}^{1/6}$$

$$\text{Nu} = 64.7$$

or a value about 8 percent higher.

7-6 | FREE CONVECTION FROM HORIZONTAL PLATES

Isothermal Surfaces

The average heat-transfer coefficient from horizontal flat plates is calculated with Equation (7-25) and the constants given in Table 7-1. The characteristic dimension for use with these relations has traditionally [4] been taken as the length of a side for a square, the mean of the two dimensions for a rectangular surface, and $0.9d$ for a circular disk. References 52 and 53 indicate that better agreement with experimental data can be achieved by calculating the characteristic dimension with

$$L = \frac{A}{P} \qquad \textbf{[7-39]}$$

where A is the area and P is the perimeter of the surface. This characteristic dimension is also applicable to unsymmetrical planforms.

Constant Heat Flux

The experiments of Reference 44 have produced the following correlations for constant heat flux on a horizontal plate. For the heated surface facing upward,

$$\overline{\text{Nu}}_L = 0.13(\text{Gr}_L \, \text{Pr})^{1/3} \qquad \text{for Gr}_L \, \text{Pr} < 2 \times 10^8 \qquad \textbf{[7-40]}$$

and

$$\overline{\text{Nu}}_L = 0.16(\text{Gr}_L\,\text{Pr})^{1/3} \qquad \text{for } 2\times 10^8 < \text{Gr}_L\,\text{Pr} < 10^{11} \qquad \textbf{[7-41]}$$

For the heated surface facing downward,

$$\overline{\text{Nu}}_L = 0.58(\text{Gr}_L\,\text{Pr})^{1/5} \qquad \text{for } 10^6 < \text{Gr}_L\,\text{Pr} < 10^{11} \qquad \textbf{[7-42]}$$

In these equations all properties except β are evaluated at a temperature T_e defined by

$$T_e = T_w - 0.25(T_w - T_\infty)$$

and T_w is the *average* wall temperature related, as before, to the heat flux by

$$\bar{h} = \frac{q_w}{T_w - T_\infty}$$

The Nusselt number is formed as before:

$$\overline{\text{Nu}}_L = \frac{\bar{h}L}{k} = \frac{q_w L}{(T_w - T_\infty)k}$$

Section 7-7 discusses an extension of these equations to inclined surfaces.

Irregular Solids

There is no general correlation which can be applied to irregular solids. The results of Reference 77 indicate that Equation (7-25) may be used with $C = 0.775$ and $m = 0.208$ for a vertical cylinder with height equal to diameter. Nusselt and Grashof numbers are evaluated by using the diameter as characteristic length. Lienhard [78] offers a prescription that takes the characteristic length as the distance a fluid particle travels in the boundary layer and uses values of $C = 0.52$ and $m = \frac{1}{4}$ in Equation (7-25) in the laminar range. This may serve as an estimate for calculating the heat-transfer coefficient in the absence of specific information on a particular geometric shape. Bodies of unity aspect ratio are studied extensively in Reference 81.

Cube Cooling in Air — EXAMPLE 7-6

A cube, 20 cm on a side, is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer.

■ Solution

This is an irregular solid so we use the information in the last entry of Table 7-1 in the absence of a specific correlation for this geometry. The properties were evaluated in Example 7-2 as

$$\beta = 3.25\times 10^{-3} \qquad k = 0.02685$$
$$\nu = 17.47\times 10^{-6} \qquad \text{Pr} = 0.7$$

The characteristic length is the distance a particle travels in the boundary layer, which is $L/2$ along the bottom plus L along the side plus $L/2$ on the top, or $2L = 40$ cm. The Gr Pr product is thus:

$$\text{Gr}\,\text{Pr} = \frac{(9.8)(3.25\times 10^{-3})(60-10)(0.4)^3}{(17.47\times 10^{-6})^2}(0.7) = 2.34\times 10^8$$

From the last entry in Table 7-1 we find $C = 0.52$ and $n = 1/4$ and calculate the Nusselt number as

$$\text{Nu} = (0.52)(2.34\times 10^8)^{1/4} = 64.3$$

and

$$\bar{h} = \mathrm{Nu}\frac{k}{L} = \frac{(64.3)(0.02685)}{(0.4)} = 4.32\ \mathrm{W/m^2 \cdot {}^\circ C}$$

The cube has six sides so the area is $6(0.2)^2 = 0.24\ \mathrm{m}^2$ and the heat transfer is

$$q = \bar{h}A(T_w - T_\infty) = (4.32)(0.24)(60 - 10) = 51.8\ \mathrm{W}$$

7-7 | FREE CONVECTION FROM INCLINED SURFACES

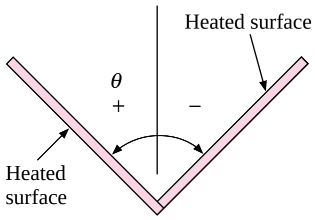


Figure 7-7 | Coordinate system for inclined plates.

Extensive experiments have been conducted by Fujii and Imura [44] for heated plates in water at various angles of inclination. The angle that the plate makes with the vertical is designated θ, with positive angles indicating that the heater surface faces downward, as shown in Figure 7-7. For the inclined plate facing downward with approximately constant heat flux, the following correlation was obtained for the average Nusselt number:

$$\overline{\mathrm{Nu}}_e = 0.56(\mathrm{Gr}_e\,\mathrm{Pr}_e \cos\theta)^{1/4} \qquad \theta < 88^\circ;\ 10^5 < \mathrm{Gr}_e\,\mathrm{Pr}_e \cos\theta < 10^{11} \qquad \textbf{[7-43]}$$

In Equation (7-43) all properties except β are evaluated at a reference temperature T_e defined by

$$T_e = T_w - 0.25(T_w - T_\infty) \qquad \textbf{[7-44]}$$

where T_w is the *mean* wall temperature and T_∞ is the free-stream temperature; β is evaluated at a temperature of $T_\infty + 0.50(T_w - T_\infty)$. For almost-horizontal plates facing downward, that is, $88^\circ < \theta < 90^\circ$, an additional relation was obtained as

$$\overline{\mathrm{Nu}}_e = 0.58(\mathrm{Gr}_e\,\mathrm{Pr}_e)^{1/5} \qquad 10^6 < \mathrm{Gr}_e\,\mathrm{Pr}_e < 10^{11} \qquad \textbf{[7-45]}$$

For an inclined plate with heated surface facing upward the empirical correlations become more complicated. For angles between -15 and -75° a suitable correlation is

$$\overline{\mathrm{Nu}}_e = 0.14[(\mathrm{Gr}_e\,\mathrm{Pr}_e)^{1/3} - (\mathrm{Gr}_c\,\mathrm{Pr}_e)^{1/3}] + 0.56(\mathrm{Gr}_e\,\mathrm{Pr}_e\cos\theta)^{1/4} \qquad \textbf{[7-46]}$$

for the range $10^5 < \mathrm{Gr}_e\ \mathrm{Pr}_e \cos\theta < 10^{11}$. The quantity Gr_c is a critical Grashof relation indicating when the Nusselt number starts to separate from the laminar relation of Equation (7-43) and is given in the following tabulation:

θ, degrees	Gr_c
-15	5×10^9
-30	2×10^9
-60	10^8
-75	10^6

For $\mathrm{Gr}_e < \mathrm{Gr}_c$ the first term of Equation (7-46) is dropped out. Additional information is given by Vliet [39] and Pera and Gebhart [45]. There is some evidence to indicate that the above relations may also be applied to constant-temperature surfaces.

Experimental measurements with air on constant-heat-flux surfaces [51] have shown that Equation (7-31) may be employed for the laminar region if we replace Gr_x^* by $\mathrm{Gr}_x^* \cos\theta$

for both upward- and downward-facing heated surfaces. In the turbulent region with air, the following empirical correlation was obtained:

$$\mathrm{Nu}_x = 0.17(\mathrm{Gr}_x^* \, \mathrm{Pr})^{1/4} \qquad 10^{10} < \mathrm{Gr}_x^* \, \mathrm{Pr} < 10^{15} \qquad \textbf{[7-47]}$$

where the Gr_x^* is the same as for the vertical plate when the heated surface faces upward. When the heated surface faces downward, Gr_x^* is replaced by $\mathrm{Gr}^* \cos^2\theta$. Equation (7-47) reduces approximately to the relation recommended in Table 7-1 for an isothermal vertical plate.

For inclined cylinders the data of Reference 73 indicate that laminar heat transfer under constant-heat-flux conditions may be calculated with the following relation:

$$\mathrm{Nu}_L = [0.60 - 0.488(\sin\theta)^{1.03}](\mathrm{Gr}_L \, \mathrm{Pr})^{\frac{1}{4}+\frac{1}{12}(\sin\theta)^{1.75}} \qquad \text{for } \mathrm{Gr}_L \, \mathrm{Pr} < 2 \times 10^8 \qquad \textbf{[7-48]}$$

where θ is the angle the cylinder makes with the vertical; that is, 0° corresponds to a vertical cylinder. Properties are evaluated at the film temperature except β, which is evaluated at ambient conditions.

Uncertainties still remain in the prediction of free convection from inclined surfaces, and an experimental-data scatter of ± 20 percent is not unusual for the empirical relations presented above.

7-8 | NONNEWTONIAN FLUIDS

When the shear-stress viscosity relation of the fluid does not obey the simple newtonian expression of Equation (5-1), the above equations for free-convection heat transfer do not apply. Extremely viscous polymers and lubricants are examples of fluids with nonnewtonian behavior. Successful analytical and experimental studies have been carried out with such fluids, but the results are very complicated. The interested reader should consult References 48 to 50 for detailed information on this subject.

7-9 | SIMPLIFIED EQUATIONS FOR AIR

Simplified equations for the heat-transfer coefficient from various surfaces to air at atmospheric pressure and moderate temperatures are given in Table 7-2. These relations may be extended to higher or lower pressures by multiplying by the following factors:

$$\left(\frac{p}{101.32}\right)^{1/2} \qquad \text{for laminar cases}$$

$$\left(\frac{p}{101.32}\right)^{2/3} \qquad \text{for turbulent cases}$$

where p is the pressure in kilopascals. Due caution should be exercised in the use of these simplified relations because they are only approximations of the more precise equations stated earlier.

The reader will note that the use of Table 7-2 requires a knowledge of the value of the Grashof-Prandtl number product. This might seem to be self-defeating, in that another calculation is required. However, with a bit of experience one learns the range of Gr Pr to be expected in various geometrical-physical situations, and thus the simplified expressions can be an expedient for quick problem solving. As we have noted, they are not a substitute for the more comprehensive expressions.

Table 7-2 | Simplified equations for free convection from various surfaces to air at atmospheric pressure, adapted from Table 7-1.

Surface	Laminar, $10^4 < Gr_f Pr_f < 10^9$	Turbulent, $Gr_f Pr_f > 10^9$
Vertical plane or cylinder	$h = 1.42\left(\frac{\Delta T}{L}\right)^{1/4}$	$h = 1.31(\Delta T)^{1/3}$
Horizontal cylinder	$h = 1.32\left(\frac{\Delta T}{d}\right)^{1/4}$	$h = 1.24(\Delta T)^{1/3}$
Horizontal plate:		
Heated plate facing upward or cooled plate facing downward	$h = 1.32\left(\frac{\Delta T}{L}\right)^{1/4}$	$h = 1.52(\Delta T)^{1/3}$
Heated plate facing downward or cooled plate facing upward	$h = 0.59\left(\frac{\Delta T}{L}\right)^{1/4}$	
Heated cube; L = length of side, Area = $6L^2$	$h = 1.052\left(\frac{\Delta T}{L}\right)^{1/4}$	

where h = heat-transfer coefficient, $W/m^2 \cdot °C$
$\Delta T = T_w - T_\infty$, °C
L = vertical or horizontal dimension, m
d = diameter, m

EXAMPLE 7-7 Calculation with Simplified Relations

Compute the heat transfer for the conditions of Example 7-5 using the simplified relations of Table 7-2.

Solution

In Example 7-5 we found that a rather large pipe with a substantial temperature difference between the surface and air still had a Gr Pr product of $1.57 \times 10^8 < 10^9$, so a laminar equation is selected from Table 7-2. The heat-transfer coefficient is given by

$$h = 1.32\left(\frac{\Delta T}{d}\right)^{1/4} = 1.32\left(\frac{250 - 15}{0.3048}\right)^{1/4}$$
$$= 6.96 \text{ W/m}^2 \cdot °\text{C}$$

The heat transfer is then

$$\frac{q}{L} = (6.96)\pi(0.3048)(250 - 15) = 1.57 \text{ kW/m}$$

Note that the simplified relation gives a value approximately 4 percent higher than Equation (7-25).

7-10 | FREE CONVECTION FROM SPHERES

Yuge [5] recommends the following empirical relation for free-convection heat transfer from spheres to air:

$$\text{Nu}_f = \frac{\bar{h}d}{k_f} = 2 + 0.392\,\text{Gr}_f^{1/4} \qquad \text{for } 1 < \text{Gr}_f < 10^5 \qquad \textbf{[7-49]}$$

This equation may be modified by the introduction of the Prandtl number to give

$$\text{Nu}_f = 2 + 0.43(\text{Gr}_f\,\text{Pr}_f)^{1/4} \qquad \textbf{[7-50]}$$

Properties are evaluated at the film temperature, and it is expected that this relation would be primarily applicable to calculations for free convection in gases. However, in the absence of more specific information it may also be used for liquids. We may note that for very low values of the Grashof-Prandtl number product the Nusselt number approaches a value of 2.0. This is the value that would be obtained for pure conduction through an infinite stagnant fluid surrounding the sphere, as obtained from Table 3-1.

For higher ranges of the Rayleigh number the experiments of Amato and Tien [79] with water suggest the following correlation:

$$\mathrm{Nu}_f = 2 + 0.50(\mathrm{Gr}_f\,\mathrm{Pr}_f)^{1/4} \qquad \textbf{[7-51]}$$

for $3 \times 10^5 < \mathrm{Gr\,Pr} < 8 \times 10^8$.

Churchill [83] suggests a more general formula for spheres, applicable over a wider range of Rayleigh numbers:

$$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_d^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}} \qquad \textbf{[7-52]}$$

for $\mathrm{Ra}_d < 10^{11}$ and $\mathrm{Pr} > 0.5$.

7-11 | FREE CONVECTION IN ENCLOSED SPACES

The free-convection flow phenomena inside an enclosed space are interesting examples of very complex fluid systems that may yield to analytical, empirical, and numerical solutions. Consider the system shown in Figure 7-8, where a fluid is contained between two vertical plates separated by the distance δ. As a temperature difference $\Delta T_w = T_1 - T_2$ is impressed on the fluid, a heat transfer will be experienced with the approximate flow regions shown in Figure 7-9, according to MacGregor and Emery [18]. In this figure, the Grashof number

Figure 7-8 | Nomenclature for free convection in enclosed vertical spaces.

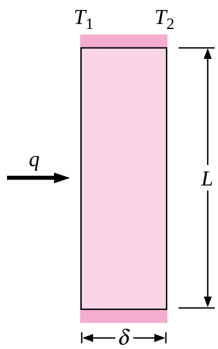
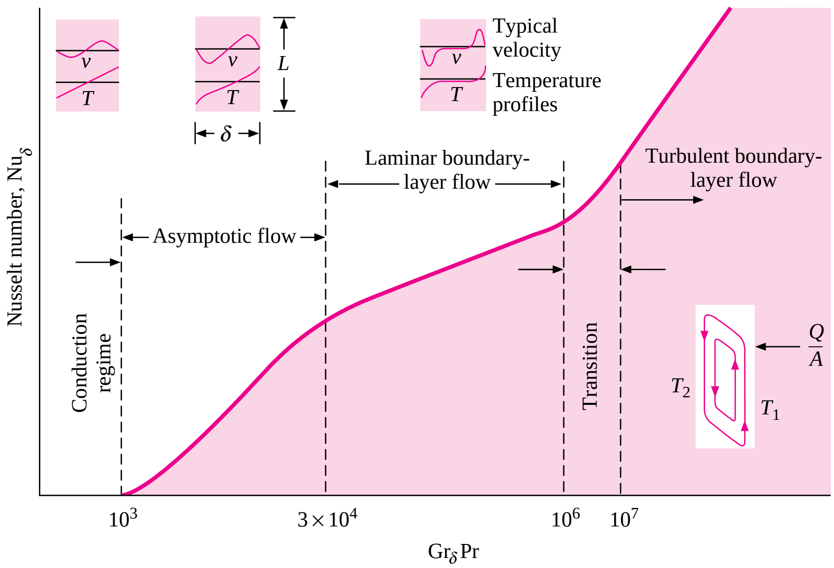


Figure 7-9 | Schematic diagram and flow regimes for the vertical convection layer, according to Reference 18.



is calculated as

$$\mathrm{Gr}_\delta = \frac{g\beta(T_1 - T_2)\delta^3}{\nu^2} \quad \textbf{[7-53]}$$

At very low Grashof numbers, there are very minute free-convection currents and the heat transfer occurs mainly by conduction across the fluid layer. As the Grashof number is increased, different flow regimes are encountered, as shown, with a progressively increasing heat transfer as expressed through the Nusselt number

$$\mathrm{Nu}_\delta = \frac{h\delta}{k}$$

Although some open questions still remain, the experiments of Reference 18 may be used to predict the heat transfer to a number of liquids under constant-heat-flux conditions. The empirical correlations obtained were:

$$\mathrm{Nu}_\delta = 0.42(\mathrm{Gr}_\delta\,\mathrm{Pr})^{1/4}\mathrm{Pr}^{0.012}\left(\frac{L}{\delta}\right)^{-0.30} \qquad q_w = \text{const} \quad \textbf{[7-54]}$$

$$10^4 < \mathrm{Gr}_\delta\,\mathrm{Pr} < 10^7$$
$$1 < \mathrm{Pr} < 20{,}000$$
$$10 < L/\delta < 40$$

$$\mathrm{Nu}_\delta = 0.46\ (\mathrm{Gr}_\delta\,\mathrm{Pr})^{1/3} \qquad q_w = \text{const} \quad \textbf{[7-55]}$$

$$10^6 < \mathrm{Gr}_\delta\,\mathrm{Pr} < 10^9$$
$$1 < \mathrm{Pr} < 20$$
$$1 < L/\delta < 40$$

The heat flux is calculated as

$$\frac{q}{A} = q_w = h(T_1 - T_2) = Nu_\delta \frac{k}{\delta}(T_1 - T_2) \quad \textbf{[7-56]}$$

The results are sometimes expressed in the alternate form of an *effective* or *apparent thermal conductivity* k_e, defined by

$$\frac{q}{A} = k_e \frac{T_1 - T_2}{\delta} \quad \textbf{[7-57]}$$

By comparing Equations (7-56) and (7-57), we see that

$$\mathrm{Nu}_\delta \equiv \frac{k_e}{k} \quad \textbf{[7-58]}$$

In the building industry the heat transfer across an air gap is sometimes expressed in terms of the R values (see Section 2-3), so that

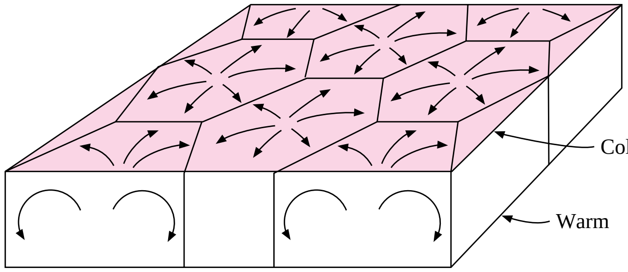
$$\frac{q}{A} = \frac{\Delta T}{R}$$

In terms of the above discussion, the R value would be

$$R = \frac{\delta}{k_e} \quad \textbf{[7-59]}$$

Heat transfer in horizontal enclosed spaces involves two distinct situations. If the upper plate is maintained at a higher temperature than the lower plate, the lower-density fluid is above the higher-density fluid and no convection currents will be experienced. In this case

Figure 7-10 | Benard-cell pattern in enclosed fluid layer heated from below, from Reference 33.



the heat transfer across the space will be by conduction alone and $\text{Nu}_\delta = 1.0$, where δ is still the separation distance between the plates. The second, and more interesting, case is experienced when the lower plate has a higher temperature than the upper plate. For values of Gr_δ below about 1700, pure conduction is still observed and $\text{Nu}_\delta = 1.0$. As convection begins, a pattern of hexagonal cells is formed as shown in Figure 7-10. These patterns are called Benard cells [33]. Turbulence begins at about $\text{Gr}_\delta = 50{,}000$ and destroys the cellular pattern.

Free convection in inclined enclosures is discussed by Dropkin and Somerscales [12]. Evans and Stefany [9] have shown that transient natural-convection heating or cooling in closed vertical or horizontal cylindrical enclosures may be calculated with

$$\text{Nu}_f = 0.55(\text{Gr}_f\,\text{Pr}_f)^{1/4} \qquad \textbf{[7-60]}$$

for the range $0.75 < L/d < 2.0$. The Grashof number is formed with the length of the cylinder L.

The analysis and experiments of Reference 43 indicate that it is possible to represent the effective thermal conductivity for fluids between concentric spheres with the relation

$$\frac{k_e}{k} = 0.228(\text{Gr}_\delta\,\text{Pr})^{0.226} \qquad \textbf{[7-61]}$$

where now the gap spacing is $\delta = r_o - r_i$. The effective thermal conductivity given by Equation (7-61) is to be used with the conventional relation for steady-state conduction in a spherical shell:

$$q = \frac{4\pi k_e r_i r_o \Delta T}{r_o - r_i} \qquad \textbf{[7-62]}$$

Equation (7-61) is valid for $0.25 \le \delta/r_i \le 1.5$ and

$$1.2 \times 10^2 < \text{Gr Pr} < 1.1 \times 10^9 \qquad 0.7 < \text{Pr} < 4150$$

Properties are evaluated at a volume mean temperature T_m defined by

$$T_m = \frac{(r_m^3 - r_i^3)T_i + (r_o^3 - r_m^3)T_o}{r_o^3 - r_i^3} \qquad \textbf{[7-63]}$$

where $r_m = (r_i + r_o)/2$. Equation (7-61) may also be used for eccentric spheres with a coordinate transformation as described in Reference 43.

Experimental results for free convection in enclosures are not always in agreement, but we can express them in a general form as

$$\frac{k_e}{k} = C(\text{Gr}_\delta\,\text{Pr})^n \left(\frac{L}{\delta}\right)^m \qquad \textbf{[7-64]}$$

Table 7-3 | Summary of empirical relations for free convection in enclosures in the form of Equation (7-61), correlation constants adjusted by Holman [74].

Fluid	Geometry	Gr_δ Pr	Pr	$\frac{L}{\delta}$	C	n	m	Reference(s)
Gas	Vertical plate,	< 2000	$k_e/k = 1.0$					6, 7, 55, 59
	isothermal	6000–200,000	0.5–2	11–42	0.197	$\frac{1}{4}$	$-\frac{1}{9}$	
		$200{,}000\text{–}1.1 \times 10^7$	0.5–2	11–42	0.073	$\frac{1}{3}$	$-\frac{1}{9}$	
	Horizontal plate,	< 1700	$k_e/k = 1.0$					
	isothermal heated from	1700–7000	0.5–2	—	0.059	0.4	0	6, 7, 55, 59, 62, 63
	below	$7000\text{–}3.2 \times 10^5$	0.5–2	—	0.212	$\frac{1}{4}$	0	66
		$> 3.2 \times 10^5$	0.5–2	—	0.061	$\frac{1}{3}$	0	
Liquid	Vertical plate,	< 2000	$k_e/k = 1.0$					
	constant heat	$10^4\text{–}10^7$	1–20,000	10–40	Eq. 7-52	—	—	18, 61
	flux or isothermal	$10^6\text{–}10^9$	1–20	1–40	0.046	$\frac{1}{3}$	0	
	Horizontal plate,	< 1700	$k_e/k = 1.0$	—				7, 8, 58, 63, 66
	isothermal,	1700–6000	1–5000	—	0.012	0.6	0	
	heated from	6000–37,000	1–5000	—	0.375	0.2	0	
	below	$37{,}000\text{–}10^8$	1–20		0.13	0.3	0	
		$> 10^8$	1–20		0.057	$\frac{1}{3}$	0	
Gas or liquid	Vertical annulus	Same as vertical plates						
	Horizontal annulus,	$6000\text{–}10^6$	1–5000	—	0.11	0.29	0	56, 57, 60
	isothermal	$10^6\text{–}10^8$	1–5000	—	0.40	0.20	0	
	Spherical annulus	$120\text{–}1.1 \times 10^9$	0.7–4000	—	0.228	0.226	0	43

Table 7-3 lists values of the constants C, n, and m for a number of physical circumstances. These values may be used for design purposes in the absence of specific data for the geometry or fluid being studied. We should remark that some of the data correlations represented by Table 7-3 have been artificially adjusted by Holman [74] to give the characteristic exponents of $\frac{1}{4}$ and $\frac{1}{3}$ for the laminar and turbulent regimes of free convection. However, it appears that the error introduced by this adjustment is not significantly greater than the disagreement between different experimental investigations. The interested reader may wish to consult the specific references for more details.

For the annulus space the heat transfer is based on

$$q = \frac{2\pi k_e L \Delta T}{\ln(r_o/r_i)} \qquad \textbf{[7-65]}$$

where L is the length of the annulus and the gap spacing is $\delta = r_o - r_i$.

Extensive correlations for free convection between cylindrical, cubical, and spherical bodies and various enclosure geometries are given by Warrington and Powe [80]. The correlations cover a wide range of fluids.

Free convection through vertical plane layers of nonnewtonian fluids is discussed in Reference 38, but the results are too complicated to present here.

In the absence of more specific design information, the heat transfer for inclined enclosures may be calculated by substituting g' for g in the Grashof number, where

$$g' = g\cos\theta \qquad \textbf{[7-66]}$$

and θ is the angle that the heater surface makes with the horizontal. This transformation may be expected to hold up to inclination angles of 60° and applies *only* to those cases where the hotter surface is facing upward. Further information is available from Hollands et al. [66, 67, 69, 82].

Radiation *R*-Value for a Gap

As we have seen in conduction heat transfer, radiation boundary conditions may play an important role in the overall heat-transfer problem. This is particularly true in free-convection situations because free-convection heat-transfer rates are typically small. We will show in Section 8-7, Equation (8-42), that the radiant transfer across a gap separating two large parallel planes may be calculated with

$$q/A = \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/\epsilon_1 + 1/\epsilon_2 - 1} \qquad \text{[7-67]}$$

where the temperatures are in degrees Kelvin and the ϵ's are the respective emissivities of the surfaces. Using the concept of the R-value discussed in Section 2-3, we could write

$$(q/A)_{\text{rad}} = \Delta T/R_{\text{rad}}$$

and thus could determine an R-value for the radiation heat transfer in conjunction with Equation (7-67). That value would be strongly temperature-dependent and would operate in parallel with the R-value for the convection across the space, which could be obtained from

$$(q/A)_{\text{conv}} = k_e \Delta T/\delta = \Delta T/R_{\text{conv}}$$

so that

$$R_{\text{conv}} = \delta/k_e$$

The total R-value for the combined radiation and convection across the space would be written as

$$R_{\text{tot}} = \frac{1}{1/R_{\text{rad}} + 1/R_{\text{conv}}}$$

The concept of combined radiation and convection in confined spaces is important in building applications.

Heat Transfer Across Vertical Air Gap — EXAMPLE 7-8

Air at atmospheric pressure is contained between two 0.5-m-square vertical plates separated by a distance of 15 mm. The temperatures of the plates are 100 and 40°C, respectively. Calculate the free-convection heat transfer across the air space. Also calculate the radiation heat transfer across the air space if both surfaces have $\epsilon = 0.2$.

■ Solution

We evaluate the air properties at the mean temperature between the two plates:

$$T_f = \frac{100 + 40}{2} = 70°\text{C} = 343 \text{ K}$$

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(343)} = 1.029 \text{ kg/m}^3$$

$$\beta = \frac{1}{T_f} = \frac{1}{343} = 2.915 \times 10^{-3} \text{ K}^{-1}$$

$$\mu = 2.043 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad k = 0.0295 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7$$

The Grashof-Prandtl number product is now calculated as

$$\mathrm{Gr}_\delta \,\mathrm{Pr} = \frac{(9.8)(1.029)^2(2.915\times10^{-3})(100-40)(15\times10^{-3})^3}{(2.043\times10^{-5})^2}0.7$$
$$= 1.027\times10^4$$

We may now use Equation (7-64) to calculate the effective thermal conductivity, with $L = 0.5$ m, $\delta = 0.015$ m, and the constants taken from Table 7-3:

$$\frac{k_e}{k} = (0.197)(1.027\times10^4)^{1/4}\left(\frac{0.5}{0.015}\right)^{-1/9} = 1.343$$

The heat transfer may now be calculated with Equation (7-54). The area is $(0.5)^2 = 0.25$ m^2, so that

$$q = \frac{(1.343)(0.0295)(0.25)(100-40)}{0.015} = 39.62 \text{ W} \qquad [135.2 \text{ Btu/h}]$$

The radiation heat flux is calculated with Equation (7-67), taking $T_1 = 373$ K, $T_2 = 313$ K, and $\epsilon_1 = \epsilon_2 = 0.2$. Thus, with $\sigma = 5.669\times10^{-8}$ W/m$^2\cdot$K^4,

$$(q/A)_{\text{rad}} = \frac{(5.669\times10^{-8})(373^4-313^4)}{[1/0.2+1/0.2-1]} = 61.47 \text{ W/m}^2$$

and

$$q_{\text{rad}} = (0.5)^2(61.47) = 15.37 \text{ W}$$

or about half the value of the convection transfer across the space. Further calculation would show that for a smaller value of $\epsilon = 0.05$, the radiation transfer is reduced to 3.55 W or, for a larger value of $\epsilon = 0.8$, the transfer is 92.2 W. In any event, radiation heat transfer can be an important factor in such problems.

EXAMPLE 7-9 Heat Transfer Across Horizontal Air Gap

Two horizontal plates 20 cm on a side are separated by a distance of 1 cm with air at 1 atm in the space. The temperatures of the plates are 100°C for the lower and 40°C for the upper plate. Calculate the heat transfer across the air space.

■ Solution

The properties are the same as given in Example 7-8:

$$\rho = 1.029 \text{ kg/m}^3 \qquad \beta = 2.915\times10^{-3}\text{ K}^{-1}$$
$$\mu = 2.043\times10^{-5}\text{ kg/m}\cdot\text{s} \qquad k = 0.0295 \text{ W/m}\cdot{}^\circ\text{C}$$
$$\mathrm{Pr} = 0.7$$

The Gr Pr product is evaluated on the basis of the separating distance, so we have

$$\mathrm{Gr}\,\mathrm{Pr} = \frac{(9.8)(1.029)^2(2.915\times10^{-3})(100-40)(0.01)^3}{(2.043\times10^{-5})^2}(0.7) = 3043$$

Consulting Table 7-3, we find $C = 0.059$, $n = 0.4$, and $m = 0$ so that

$$\frac{k_e}{k} = (0.059)(3043)^{0.4}\left(\frac{0.2}{0.01}\right)^0 = 1.46$$

and

$$q = \frac{k_e A(T_1-T_2)}{\delta} = \frac{(1.460)(0.0295)(0.2)^2(100-40)}{0.01} = 10.34 \text{ W}$$

Heat Transfer Across Water Layer

EXAMPLE 7-10

Two 50-cm horizontal square plates are separated by a distance of 1 cm. The lower plate is maintained at a constant temperature of 100°F and the upper plate is constant at 80°F. Water at atmospheric pressure occupies the space between the plates. Calculate the heat lost by the lower plate.

■ Solution

We evaluate properties at the mean temperature of 90°F and obtain, for water,

$$k = 0.623 \text{ W/m} \cdot {}^\circ\text{C} \qquad \frac{g\beta\rho^2 c_p}{\mu k} = 2.48 \times 10^{10}$$

The Grashof-Prandtl number product is now evaluated using the plate spacing of 1 cm as the characteristic dimension.

$$\text{Gr Pr} = (2.48 \times 10^{10})(0.01)^3(100 - 80)(5/9) = 2.76 \times 10^5$$

Now, using Equation (7-64) and consulting Table 7-3 we obtain

$$C = 0.13 \qquad n = 0.3 \qquad m = 0$$

Therefore, Equation (7-64) becomes

$$\frac{k_e}{k} = (0.13)(2.76 \times 10^5)^{0.3} = 5.57$$

The effective thermal conductivity is thus

$$k_e = (0.623)(5.57) = 3.47 \text{ W/m} \cdot {}^\circ\text{C}$$

and the heat transfer is

$$q = k_e A \Delta T/\delta = \frac{(3.47)(0.5)^2(100 - 80)(5/9)}{0.01} = 964 \text{ W}$$

We see, of course, that the heat transfer across a water gap is considerably larger than for an air gap [Example 7-9] because of the larger thermal conductivity.

Reduction of Convection in Air Gap

EXAMPLE 7-11

A vertical air gap between two glass plates is to be evacuated so that the convective currents are essentially eliminated, that is, the air behaves as a pure conductor. For air at a mean temperature of 300 K and a temperature difference of 20°C, calculate the vacuum necessary for glass spacings of 1 and 2 cm.

■ Solution

Consulting Table 7-3, we find that for gases, a value of $\text{Gr}_\delta \text{ Pr} < 2000$ is necessary to reduce the system to one of pure conduction. At 300 K the propertiues of air are

$$k = 0.02624 \text{ W/m} \cdot {}^\circ\text{C} \quad \text{Pr} = 0.7 \quad \mu = 1.846 \times 10^{-5} \text{ kg/m} \cdot \text{s} \quad \beta = 1/300$$

and

$$\rho = p/RT = p/(287)(300)$$

We have

$$\begin{aligned} \text{Gr}_\delta \text{ Pr} &= g\beta\rho^2 \Delta T \delta^3 \text{Pr}/\mu^2 = 2000 \\ &= (9.8)(1/300)[p/(287)(300)]^2(20)\delta^3(0.7)/(1.846 \times 10^{-5})^2 \end{aligned}$$

and $p^2\delta^3 = 7773$. Therefore, for a plate spacing of $\delta = 1$ cm we have

$$p = [7773/(0.01)^3]^{1/2} = 88200 \text{ Pa}$$

or, vacuum $= p_{atm} - p = 101320 - 88200 = 13120$ Pa. For a spacing of 2 cm,

$$p = 31190 \text{ Pa and vacuum} = 70130 \text{ Pa}$$

Both vacuum figures are modest and easily achieved in practice.

Evacuated (Low-Density) Spaces

In the equations presented for free convection in enclosures we have seen that when the product $\text{Gr}_\delta \text{Pr}$ is sufficiently small, usually less than about 2000, the fluid layer behaves as if pure conduction were involved and $k_e/k \to 1.0$. This means that the free-convection flow velocities are small. A small value of Gr_δ can result from either lowering the fluid pressure (density) or by reducing the spacing δ. If the pressure of a gas is reduced sufficiently, we refer to the situation as a low-density problem, which is influenced by the mean free path of the molecules and by individual molecular impacts.

A number of practical situations involve heat transfer between a solid surface and a low-density gas. In employing the term *low density*, we shall mean those circumstances where the mean free path of the gas molecules is no longer small in comparison with a characteristic dimension of the heat-transfer surface. The *mean free path* λ is the distance a molecule travels, on the average, between collisions. The larger this distance becomes, the greater the distance required to communicate the temperature of a hot surface to a gas in contact with it. This means that we shall not necessarily be able to assume that a gas in the immediate neighborhood of the surface will have the same temperature as the heated surface, as was done in the boundary-layer analyses. Evidently, the parameter that is of principal interest is a ratio of the mean free path to a characteristic body dimension. This grouping is called the Knudsen number,

$$\text{Kn} = \frac{\lambda}{L} \qquad \textbf{[7-68]}$$

According to the kinetic theory of gases, the mean free path may be calculated from

$$\lambda = \frac{0.707}{4\pi r^2 n} \qquad \textbf{[7-69]}$$

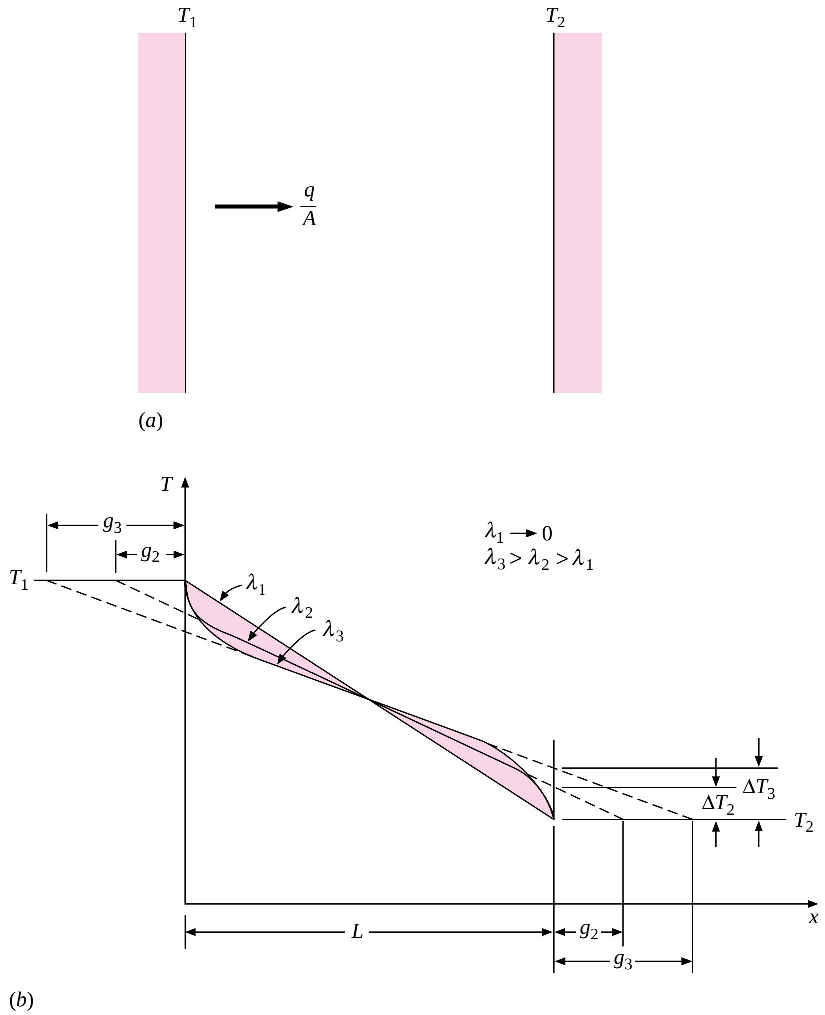
where r is the effective molecular radius for collisions and n is the molecular density. An approximate relation for the mean free path of air molecules is given by

$$\lambda = 2.27 \times 10^{-5} \frac{T}{p} \text{ meters} \qquad \textbf{[7-70]}$$

where T is in degrees Kelvin and p is in pascals.

As a first example of low-density heat transfer let us consider the two parallel infinite plates shown in Figure 7-11. The plates are maintained at different temperatures and separated by a gaseous medium. Let us first consider a case where the density or plate spacing is low enough that free convection effects are negligible, but with a gas density sufficiently high so that $\lambda \to 0$ and a linear temperature profile through the gas will be

Figure 7-11 | Effect of mean free path on conduction heat transfer between parallel plates: (*a*) physical model; (*b*) anticipated temperature profiles.



experienced, as shown for the case of λ_1. As the gas density is lowered, the larger mean free paths require a greater distance from the heat-transfer surfaces in order for the gas to accommodate to the surface temperatures. The anticipated temperature profiles are shown in Figure 7-11*b*. Extrapolating the straight portion of the low-density curves to the wall produces a temperature "jump" ΔT, which may be calculated by making the following energy balance:

$$\frac{q}{A} = k\frac{T_1 - T_2}{g + L + g} = k\frac{\Delta T}{g} \quad \textbf{[7-71]}$$

In this equation we are assuming that the extrapolation distance g is the same for both plate surfaces. In general, the temperature jump will depend on the type of surface, and these extrapolation distances will not be equal unless the materials are identical. For different types of materials we should have

$$\frac{q}{A} = k\frac{T_1 - T_2}{g_1 + L + g_2} = k\frac{\Delta T_1}{g_1} = k\frac{\Delta T_2}{g_2} \qquad \textbf{[7-72]}$$

where now ΔT_1 and ΔT_2 are the temperature jumps at the two heat-transfer surfaces and g_1 and g_2 are the corresponding extrapolation distances. For identical surfaces the temperature jump would then be expressed as

$$\Delta T = \frac{g}{2g + L}(T_1 - T_2) \qquad \textbf{[7-73]}$$

Similar expressions may be developed for low-density conduction between concentric cylinders. In order to predict the heat-transfer rate it is necessary to establish relations for the temperature jump for various gas-to-solid interfaces.

We have already mentioned that the temperature-jump effect arises as a result of the failure of the molecules to "accommodate" to the surface temperature when the mean free path becomes of the order of a characteristic body dimension. The parameter that describes this behavior is called the *accommodation coefficient* α, defined by

$$\alpha = \frac{E_i - E_r}{E_i - E_w} \qquad \textbf{[7-74]}$$

where

E_i = energy of incident molecules on a surface
E_r = energy of molecules reflected from the surface
E_w = energy molecules would have if they acquired energy of wall at temperature T_w

Values of the accommodation coefficient must be determined from experiment, and some typical values are given in Table 7-4.

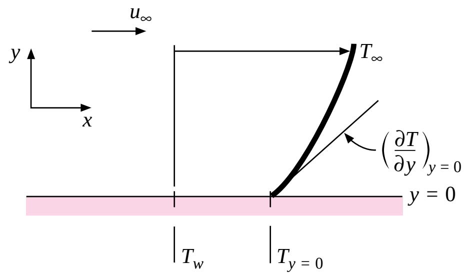
It is possible to employ the kinetic theory of gases along with values of α to determine the temperature jump at a surface. The result of such an analysis is

$$T_{y=0} - T_w = \frac{2-\alpha}{\alpha}\frac{2\gamma}{\gamma+1}\frac{\lambda}{\mathrm{Pr}}\frac{\partial T}{\partial y}\bigg]_{y=0} \qquad \textbf{[7-75]}$$

Table 7-4 | Thermal accommodation coefficients for air at low pressure in contact with various surfaces.

Surface	Accommodation coefficient, α
Flat black lacquer on bronze	0.88–0.89
Bronze, polished	0.91–0.94
Machined	0.89–0.93
Etched	0.93–0.95
Cast iron, polished	0.87–0.93
Machined	0.87–0.88
Etched	0.89–0.96
Aluminum, polished	0.87–0.95
Machined	0.95–0.97
Etched	0.89–0.97

Figure 7-12 | Nomenclature for use with Equation (7-75).



The nomenclature for Equation (7-75) is noted in Figure 7-12. This temperature jump is denoted by ΔT in Figure 7-11, and the temperature gradient for use with Figure 7-11 would be

$$\frac{T_1 - T_2 - 2\Delta T}{L}$$

For very low densities (high vacuum) the mean free path may become very large compared to the plate separation distance and the conduction-convection heat transfer will approach zero. The reader should recognize, however, that the total heat transfer across the gap-space will be the sum of conduction-convection and radiation heat transfer. We will discuss radiation heat transfer in detail in Chapter 8, but we have already provided the relation in Equation (7-67) for calculation of radiant heat transfer between two parallel plates. We note that ϵ approaches 1.0 for highly absorptive surfaces and has a small value for highly reflective surfaces. Example 7-12 illustrates the application of the low-density relations to calculation of heat transfer across a gap.

Heat Transfer Across Evacuated Space — EXAMPLE 7-12

Two polished-aluminum plates ($\epsilon = 0.06$) are separated by a distance of 2.5 cm in air at a pressure of 10^{-6} atm. The plates are maintained at 100 and 30°C, respectively. Calculate the conduction heat transfer through the air gap. Compare this with the radiation heat transfer and the conduction for air at normal atmospheric pressure.

■ Solution

We first calculate the mean free path to determine if low-density effects are important. From Equation (7-70), at an average temperature of 65°C = 338 K,

$$\lambda = \frac{(2.27 \times 10^{-5})(338)}{(1.0132 \times 10^{+5})(10^{-6})} = 0.0757 \text{ m} = 7.57 \text{ cm} \quad [0.248 \text{ ft}]$$

Since the plate spacing is only 2.5 cm, we should expect low-density effects to be important. Evaluating properties at the mean air temperature of 65°C, we have

$$k = 0.0291 \text{ W/m} \cdot {}^\circ\text{C} \quad [0.0168 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}]$$

$$\gamma = 1.40 \qquad \text{Pr} = 0.7 \qquad \alpha \approx 0.9 \qquad \text{from Table 7-4}$$

Combining Equation (7-75) with the central-temperature-gradient relation gives

$$\Delta T = \frac{2 - \alpha}{\alpha} \frac{2\gamma}{\gamma + 1} \frac{\lambda}{\text{Pr}} \frac{T_1 - T_2 - 2\Delta T}{L}$$

Inserting the appropriate properties gives

$$\Delta T = \frac{2-0.9}{0.9}\frac{2.8}{2.4}\frac{0.0757}{0.7}\frac{100-30-2\Delta T}{0.025}$$
$$= 32.38°\text{C} \qquad [58.3°\text{F}]$$

The conduction heat transfer is thus

$$\frac{q}{A} = k\frac{T_1 - T_2 - 2\Delta T}{L} = \frac{(0.0291)(70-64.76)}{0.025}$$
$$= 6.099 \text{ W/m}^2 \quad [1.93 \text{ Btu/h} \cdot \text{ft}^2]$$

At normal atmospheric pressure the conduction would be

$$\frac{q}{A} = k\frac{T_1 - T_2}{L} = 81.48 \text{ W/m}^2 \quad [25.8 \text{ Btu/h} \cdot \text{ft}^2]$$

The radiation heat transfer is calculated with Equation (8-42), taking $\epsilon_1 = \epsilon_2 = 0.06$ for polished aluminum:

$$\left(\frac{q}{A}\right)_{\text{rad}} = \frac{\sigma(T_1^4 - T_2^4)}{2/\epsilon - 1} = \frac{(5.669 \times 10^{-8})(393^4 - 303^4)}{2/0.06 - 1}$$
$$= 27.05 \text{ W/m}^2 \quad [8.57 \text{ Btu/h} \cdot \text{ft}^2]$$

Thus, at the low-density condition the radiation heat transfer is almost 5 times as large as the conduction, even with highly polished surfaces.

7-12 | COMBINED FREE AND FORCED CONVECTION

A number of practical situations involve convection heat transfer that is neither "forced" nor "free" in nature. The circumstances arise when a fluid is forced over a heated surface at a rather low velocity. Coupled with the forced-flow velocity is a convective velocity that is generated by the buoyancy forces resulting from a reduction in fluid density near the heated surface.

A summary of combined free- and forced-convection effects in tubes has been given by Metais and Eckert [10], and Figure 7-13 presents the regimes for combined convection in vertical tubes. Two different combinations are indicated in this figure. *Aiding flow* means that the forced- and free-convection currents are in the same direction, while *opposing flow* means that they are in the opposite direction. The abbreviation UWT means uniform wall temperature, and the abbreviation UHF indicates data for uniform heat flux. It is fairly easy to anticipate the qualitative results of the figure. A large Reynolds number implies a large forced-flow velocity, and hence less influence of free-convection currents. The larger the value of the Grashof-Prandtl product, the more one would expect free-convection effects to prevail.

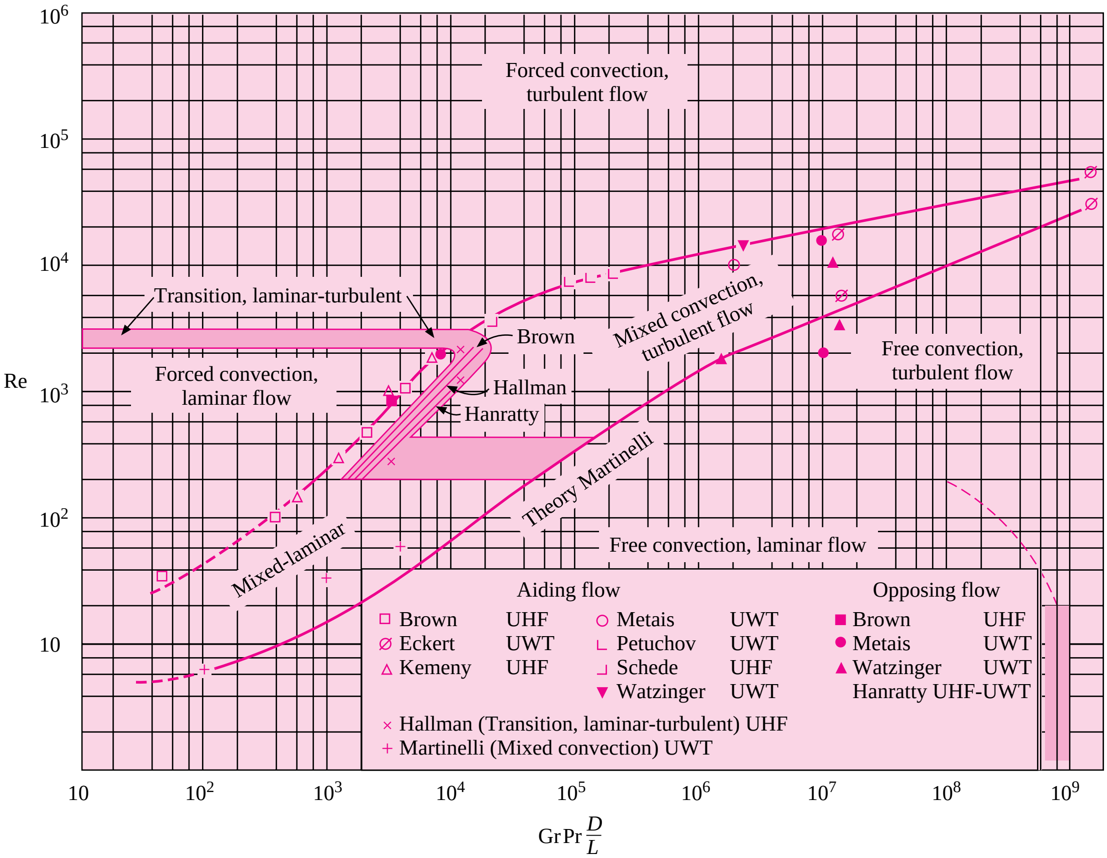
Figure 7-14 presents the regimes for combined convection in horizontal tubes. In this figure the Graetz number is defined as

$$\text{Gz} = \text{Re}\,\text{Pr}\frac{d}{L} \qquad \textbf{[7-76]}$$

The applicable range of Figures 7-13 and 7-14 is for

$$10^{-2} < \text{Pr}\left(\frac{d}{L}\right) < 1$$

Figure 7-13 | Regimes of free, forced, and mixed convection for flow through vertical tubes, according to Reference 10.



The correlations presented in the figures are for constant wall temperature. All properties are evaluated at the film temperature.

Brown and Gauvin [17] have developed a better correlation for the mixed-convection, laminar flow region of Figure 7-14:

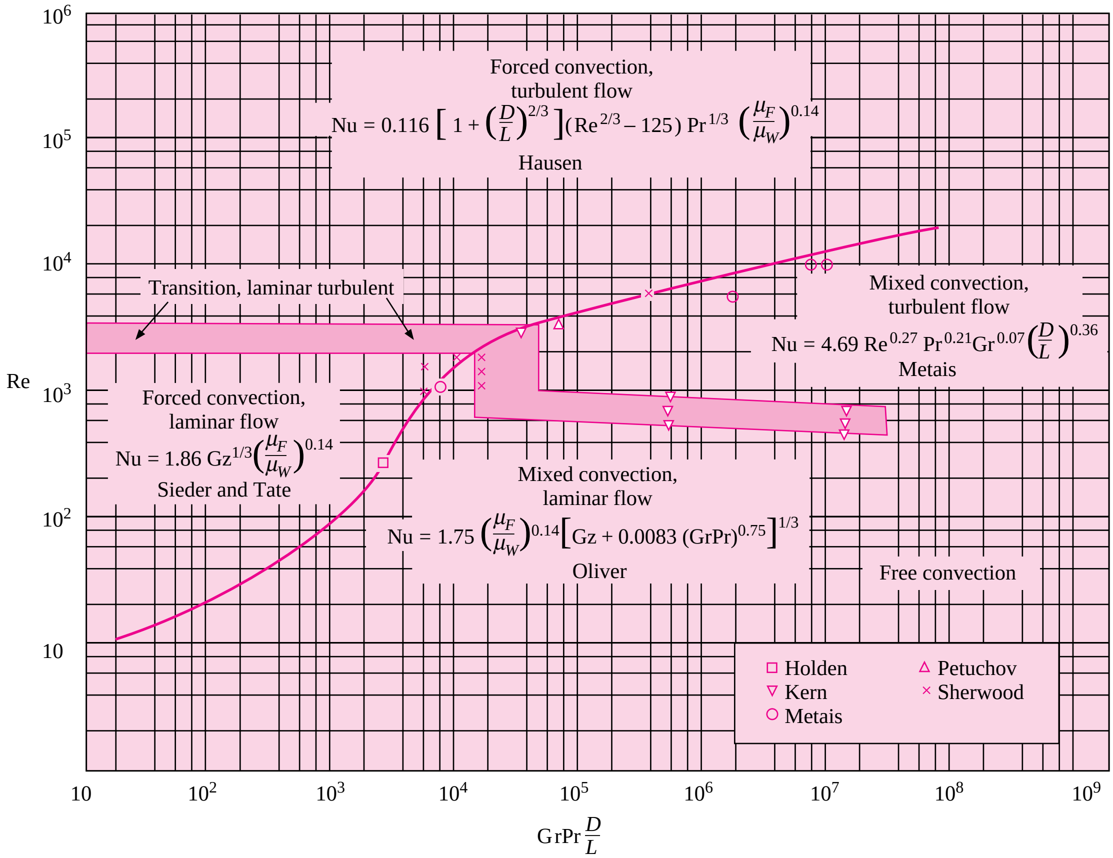
$$\mathrm{Nu} = 1.75\left(\frac{\mu_b}{\mu_w}\right)^{0.14}[\mathrm{Gz} + 0.012(\mathrm{Gz}\,\mathrm{Gr}^{1/3})^{4/3}]^{1/3} \qquad \textbf{[7-77]}$$

where μ_b is evaluated at the bulk temperature. This relation is preferred over that shown in Figure 7-14. Further information is available in Reference 68. The problem of combined free and forced convection from horizontal cylinders is treated in detail by Fand and Keswani [47].

Criterion for Free or Forced Convection

The general notion that is applied in combined-convection analysis is that the predominance of a heat-transfer mode is governed by the fluid velocity associated with that mode. A forced-convection situation involving a fluid velocity of 30 m/s, for example, would be expected to overshadow most free-convection effects encountered in ordinary gravitational fields because the velocities of the free-convection currents are small in comparison with

Figure 7-14 | Regimes of free, forced, and mixed convection for flow through horizontal tubes, according to Metais and Eckert [10].



30 m/s. On the other hand, a forced-flow situation at very low velocities (~ 0.3 m/s) might be influenced appreciably by free-convection currents. An order-of-magnitude analysis of the free-convection boundary-layer equations will indicate a general criterion for determining whether free-convection effects dominate. The criterion is that when

$$\mathrm{Gr}/\mathrm{Re}^2 > 10 \qquad \textbf{[7-78]}$$

free convection is of primary importance. This result is in agreement with Figures 7-13 and 7-14.

EXAMPLE 7-13 Combined Free and Forced Convection with Air

Air at 1 atm and 27°C is forced through a horizontal 25-mm-diameter tube at an average velocity of 30 cm/s. The tube wall is maintained at a constant temperature of 140°C. Calculate the heat-transfer coefficient for this situation if the tube is 0.4 m long.

■ Solution

For this calculation we evaluate properties at the film temperature:

$$T_f = \frac{140 + 27}{2} = 83.5°\mathrm{C} = 356.5\ \mathrm{K}$$

$$\rho_f = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(356.5)} = 0.99 \text{ kg/m}^3$$

$$\beta = \frac{1}{T_f} = 2.805 \times 10^{-3} \text{ K}^{-1} \qquad \mu_w = 2.337 \times 10^{-5} \text{ kg/m} \cdot \text{s}$$

$$\mu_f = 2.102 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad k_f = 0.0305 \text{ W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 0.695$$

Let us take the bulk temperature as 27°C for evaluating μ_b; then

$$\mu_b = 1.8462 \times 10^{-5} \text{ kg/m} \cdot \text{s}$$

The significant parameters are calculated as

$$\text{Re}_f = \frac{\rho u d}{\mu} = \frac{(0.99)(0.3)(0.025)}{2.102 \times 10^{-5}} = 3.53$$

$$\text{Gr} = \frac{\rho^2 g\beta(T_w - T_b)d^3}{\mu^2} = \frac{(0.99)^2(9.8)(2.805 \times 10^{-3})(140 - 27)(0.025)^3}{(2.102 \times 10^{-5})^2}$$

$$= 1.007 \times 10^5$$

$$\text{Gr Pr}\frac{d}{L} = (1.077 \times 10^5)(0.695)\frac{0.025}{0.4} = 4677$$

According to Figure 7-14, the mixed-convection-flow regime is encountered. Thus we must use Equation (7-77). The Graetz number is calculated as

$$\text{Gz} = \text{Re Pr}\frac{d}{L} = \frac{(353)(0.695)(0.025)}{0.4} = 15.33$$

and the numerical calculation for Equation (7-77) becomes

$$\text{Nu} = 1.75\left(\frac{1.8462}{2.337}\right)^{0.14}\{15.33 + (0.012)[(15.33)(1.077 \times 10^5)^{1/3}]^{4/3}\}^{1/3}$$

$$= 7.70$$

The average heat-transfer coefficient is then calculated as

$$\bar{h} = \frac{k}{d}\text{Nu} = \frac{(0.0305)(7.70)}{0.025} = 9.40 \text{ W/m}^2 \cdot {}^\circ\text{C} \quad [1.67 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}]$$

It is interesting to compare this value with that which would be obtained for strictly laminar forced convection. The Sieder-Tate relation [Equation (6-10)] applies, so that

$$\text{Nu} = 1.86(\text{Re Pr})^{1/3}\left(\frac{\mu_f}{\mu_w}\right)^{0.14}\left(\frac{d}{L}\right)^{1/3}$$

$$= 1.86 \text{ Gz}^{1/3}\left(\frac{\mu_f}{\mu_w}\right)^{0.14}$$

$$= (1.86)(15.33)^{1/3}\left(\frac{2.102}{2.337}\right)^{0.14}$$

$$= 4.55$$

and

$$\bar{h} = \frac{(4.55)(0.0305)}{0.025} = 5.55 \text{ W/m}^2 \cdot {}^\circ\text{C} \quad [0.977 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}]$$

Thus there would be an error of −41 percent if the calculation were made strictly on the basis of laminar forced convection.

Table 7-5 | Summary of free-convection heat-transfer relations T. For most cases, properties are evaluated at $T_f = (T_w + T_\infty)/2$.

Geometry	Equation	Restrictions	Equation number
A variety of isothermal surfaces	$\mathrm{Nu}_f = C(\mathrm{Gr}_f\,\mathrm{Pr}_f)^m$ C and m from Table 7-1	See Table 7-1	(7-25)
Vertical isothermal surface	$\overline{\mathrm{Nu}}^{1/2} = 0.825 + \frac{0.387\,\mathrm{Ra}^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}$	$10^{-1} < \mathrm{Ra}_L < 10^{12}$ Also see Fig. 7-5	(7-29)
Vertical surface, constant heat flux, local h	$\mathrm{Nu}_{xf} = C(\mathrm{Gr}_x^*\mathrm{Pr}_f)^m$	$C = 0.60$, $m = \frac{1}{5}$ for $10^5 < \mathrm{Gr}_x^*\mathrm{Pr} < 10^{11}$	(7-31)
		$C = 0.17$, $m = \frac{1}{4}$ for $2 \times 10^{13} < \mathrm{Gr}^*\mathrm{Pr} < 10^{16}$	(7-32)
Isothermal horizontal cylinders	$\overline{\mathrm{Nu}}^{1/2} = 0.60 + 0.387\left\{\frac{\mathrm{Gr\,Pr}}{[1+(0.559/\mathrm{Pr})^{9/16}]^{16/9}}\right\}^{1/6}$	$10^{-5} < \mathrm{Gr\,Pr} < 10^{13}$ Also see Fig. 7-6	(7-36)
Horizontal surface, constant heat flux		See text	(7-39) to (7-42)
Inclined surfaces	Section 7-7	See text	
Spheres	$\mathrm{Nu} = 2 + 0.43(\mathrm{Gr\,Pr})^{1/4}$	$1 < \mathrm{Gr\,Pr} < 10^5$	(7-50)
	$\mathrm{Nu} = 2 + 0.5(\mathrm{Gr\,Pr})^{1/4}$	water, $3 \times 10^5 < \mathrm{Gr\,Pr} < 8 \times 10^8$	(7-51)
	$\mathrm{Nu} = 2 + \frac{0.589(\mathrm{Gr\,Pr})^{1/4}}{[1+(0.469/\mathrm{Pr})^{9/16}]^{4/9}}$	$0.5 < \mathrm{Pr}$ $\mathrm{Gr\,Pr} < 10^{11}$	(7-52)
Enclosed spaces	$q = k_e A(\Delta T/\delta)$ $\frac{k_e}{k} = C(\mathrm{Gr}_\delta\,\mathrm{Pr})^n\,(L/\delta)^m$	Constants C, m, and n from Table 7-3 Pure conduction for $\mathrm{Gr}_\delta\,\mathrm{Pr} < 2000$	(7-57) (7-64)
Across evacuated spaces	Most transfer is by radiation		

7-13 | SUMMARY

By now the reader will have sensed that there is an abundance of empirical relations for natural convection systems. Our purposes in this section are to (1) issue a few words of caution and (2) provide a convenient table to summarize the relations.

Most free-convection data are collected under laboratory conditions in still air, still water, etc. A practical free-convection problem might not be so fortunate and the boundary layer could have a slightly added forced-convection effect. In addition, real surfaces in practice are *seldom* isothermal or constant heat flux so the correlations developed from laboratory data for these conditions may not strictly apply. The net result, of course, is that the engineer must realize that calculated values of the heat-transfer coefficient can vary $\pm$ 25 percent from what will actually be experienced.

For solution of free-convection problems one should follow a procedure similar to that given in Chapter 6 for forced-convection problems. To aid the reader, a summary of free-convection correlations is given in Table 7-5.

7-14 | SUMMARY PROCEDURE FOR ALL CONVECTION PROBLEMS

At the close of Chapter 6 we gave a brief procedure for calculation of convection heat transfer. We now are in a position to expand that discussion to include the possibility of free-convection exchange. The procedure is as follows:

1. Specify the fluid involved and be prepared to determine properties of that fluid. This may seem like a trivial step, but a surprisingly large number of errors are made in practice by choosing the *wrong* fluid, that is to say, air instead of water.