

the Reynolds number is increased, the phenomenon propagates upstream. This makes the transition from one regime to the other more gradual in staggered arrays (triangles or rotated squares) than in nonstaggered arrays (squares). (See the shape of the curves in Fig. 7-27.)

The results shown in the figure correspond to a bank with 10 tube rows. These results also can be used for a higher number of rows, but the results must be corrected if the number of rows is less than 10.

4-4 NATURAL CONVECTION

4-4-1 Heat Transfer Mechanism in Natural Convection

Let's consider a body at initial temperature T_o that at a certain time is submerged into a fluid at temperature T_∞ , where $T_o > T_\infty$. Owing to the temperature difference, heat begins to flow from the body to the fluid.

The heat transfer provokes an increase in the temperature of the fluid. Thus the density of the fluid closer to the body decreases below the density of the bulk of the fluid. This density difference creates an ascendant movement of the fluid closer to the body, which is replaced by fresh fluid coming from the colder zones. The shapes of the streamlines for several geometries are shown in Fig. 4-29. This fluid movement, in turn, helps to increase the heat transfer by convection because it allows the renewal of the fluid in the vicinity of the body surface.

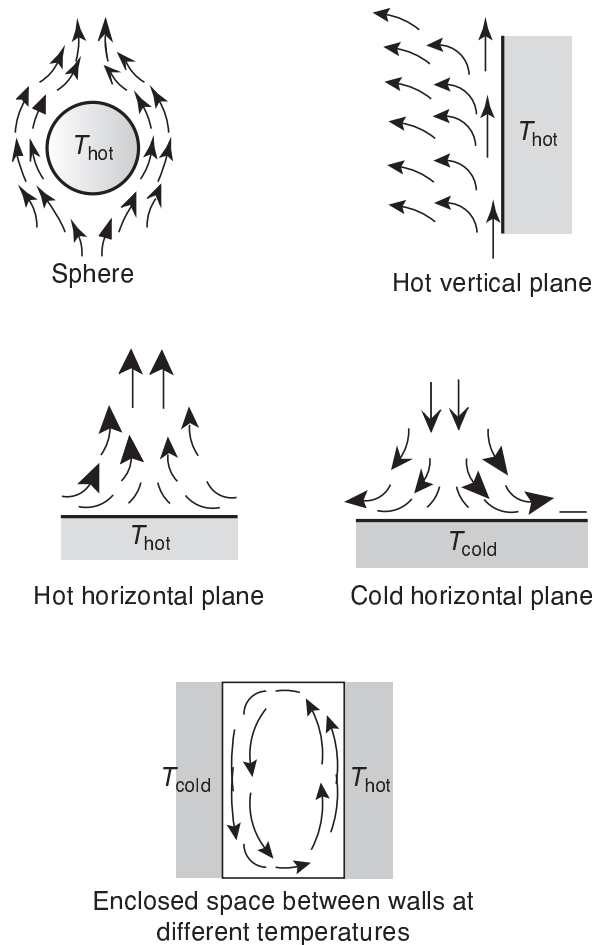


FIGURE 4-29 Streamlines in natural convection.

The difference with respect to the other cases we studied previously is that now the heat transfer is what provokes the movement of the fluid. The fluid velocity is not imposed externally. It is not a predefined variable. The velocity profile depends on the thermal characteristics of the system.

The heat transfer coefficient in natural convection depends, as always, on the fluid properties c , ρ , μ , and k . The cause of the fluid movement is the change in density created by the applied temperature difference. The physical property that indicates how the density varies with temperature is the thermal expansion coefficient β , which is defined as

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_{p=cte} \quad (4-4-1)$$

For the case of an ideal gas,

$$\rho = \frac{Mp}{RT} \quad (4-4-2)$$

Then

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p = \frac{1}{T} \quad (4-4-3)$$

We see that for an ideal gas the thermal expansion coefficient is the reciprocal of the absolute temperature. For other fluids, β must be calculated from tabulated data of densities at different temperatures.

The convection heat transfer coefficient obviously will be a function of the temperature difference between solid and fluid as well because this temperature difference is the cause of the fluid movement. We then can say

$$h = h [c, \mu, \rho, \beta, (T_o - T_\infty), k, L] \quad (4-4-4)$$

where L is a characteristic length of the geometry.

By applying dimensional analysis techniques, we can find a set of dimensionless groups to correlate the phenomenon. One possible solution is

$$\frac{hL}{k} = f \left[\frac{L^3 \rho^2 g \beta (T_o - T_\infty)}{\mu^2}, \frac{c\mu}{k} \right] \quad (4-4-5)$$

The first group within brackets is called the *Grashoff number* (Gr), that is,

$$\text{Gr} = \frac{L^3 \rho^2 g \beta (T_o - T_\infty)}{\mu^2} \quad (4-4-6)$$

The other dimensionless groups in Eq. (4-4-12) are the Nusselt and Prandtl numbers. It is observed experimentally that the correlations are usually of the type

$$\text{Nu} = a(\text{Gr} \cdot \text{Pr})^b \quad (4-4-7)$$

This means that the product $\text{Gr} \cdot \text{Pr}$ behaves as a single dimensionless group in the correlations. The physical properties included in the dimensionless groups are evaluated at the film temperature, which is the arithmetic mean value between the surface temperature and the temperature of the bulk of the fluid.

4-4-2 Correlations for Different Geometries

The correlations for the cases of higher practical interest are presented below.

Vertical Plates and Vertical Cylinders. According to Eckert and Jackson,⁶

$$\text{Nu}_L = \frac{hL}{k} = 0.555(\text{Gr}_L \cdot \text{Pr})^{1/4} \quad \text{for } \text{Gr}_L \cdot \text{Pr} < 10^9 \quad (4-4-8)$$

$$\text{Nu}_L = \frac{hL}{k} = 0.021(\text{Gr}_L \cdot \text{Pr})^{2/5} \quad \text{for } \text{Gr}_L \cdot \text{Pr} > 10^9 \quad (4-4-9)$$

In these expressions, L is the height of the plate or cylinder, and h is the average coefficient for the entire plate.

Horizontal Cylinders. According to McAdams,⁴

$$\text{Nu} = \frac{hD}{k} = 0.53(\text{Gr}_D \cdot \text{Pr})^{1/4} \quad \text{for } 10^4 < \text{Gr}_D \cdot \text{Pr} < 10^9 \quad (4-4-10)$$

$$\text{Nu} = \frac{hD}{k} = 0.114(\text{Gr}_D \cdot \text{Pr})^{1/3} \quad \text{for } 10^9 < \text{Gr}_D \cdot \text{Pr} < 10^{12} \quad (4-4-11)$$

Horizontal Planar Surfaces. According to McAdams,⁴ for hot horizontal planes facing upward or cold horizontal planes facing downward,

$$\text{Nu}_L = 0.54(\text{Gr}_L \cdot \text{Pr})^{1/4} \quad \text{for } 10^5 < \text{Gr}_L \cdot \text{Pr} < 2 \times 10^7 \quad (4-4-12)$$

$$\text{Nu}_L = 0.14(\text{Gr}_L \cdot \text{Pr})^{1/3} \quad \text{for } 2 \times 10^7 < \text{Gr}_L \cdot \text{Pr} < 3 \times 10^{10} \quad (4-4-13)$$

For hot horizontal plates facing downward and cold horizontal plates facing upward in the range of $3 \times 10^5 < \text{Gr} \cdot \text{Pr} < 10^{10}$, the recommended correlation is

$$\text{Nu}_L = 0.27(\text{Gr}_L \cdot \text{Pr})^{1/4} \quad (4-4-14)$$

In these expressions, the characteristic length is the length of one side if the surface is a square or the mean value of both sides if it is a rectangle or 0.9 times the diameter if it is a circle.

Spheres. According to Ranz and Marshall,¹³ for single spheres in an infinite fluid,

$$\text{Nu}_D = 2 + 0.60 \text{Gr}_D^{1/4} \cdot \text{Pr}^{1/3} \quad (4-4-15)$$

Example 4-3 Calculate what must be the temperature of the bath in Example 4-1 to reach the required 60°C at the tube wall.

Solution The heat flux transferred from the external fluid (hot water) to the tube wall must be the same as that transferred from the tube wall to the process fluid. Thus

$$Q = 29,564 \text{ W} = h_o A_o (T - 60)$$

where T is the temperature of the water, and A_o is the external area of the coil. An iterative procedure must be employed because h_o is also a function of the water temperature. The steps are as follows:

1. Assume T .
2. Calculate h_o .
3. Verify Q .

Let's assume the $T = 90^\circ\text{C}$. The film temperature of the water is $(90 + 60)/2 = 75^\circ\text{C}$. At this temperature, the physical properties of the water are

- Density = 974.8 kg/m^3
- Viscosity = 0.377 cP
- Thermal conductivity = $0.666 \text{ W/(m} \cdot \text{K)}$
- Specific heat = $4,190 \text{ J/(kg} \cdot \text{K)}$

To calculate the thermal expansion coefficient β , we need to know the density at other temperatures. At 80°C , the density of water is 971.8 ; thus,

$$\beta = \frac{1}{\rho} \frac{\Delta\rho}{\Delta T} = \frac{1}{973.3} \frac{974.8 - 971.8}{5} = 6.15 \times 10^{-4}$$

Then

$$\text{Gr} \cdot \text{Pr} = \left(\frac{D_o^3 \rho^2 g \beta \Delta T}{\mu^2} \frac{c\mu}{k} \right) = \left(\frac{0.0604^3 \times 974.8^2 \times 9.8 \times 6.15 \times 10^{-4} \times 30}{(0.377 \times 10^{-3})^2} \frac{4,190 \times 0.377 \times 10^{-3}}{0.666} \right) = 6.3 \times 10^8$$

$$\text{Nu} = 0.53(\text{Gr} \cdot \text{Pr})^{0.25} = 84$$

$$h_o = \text{Nu} \cdot k/D_o = 84(0.666/0.0604) = 926 \text{ W/(m}^2 \cdot \text{K)}$$

$$Q = h_o A_o (T - T_w) = 926(\pi D_o L)(90 - 60) = 926(\pi 0.0604 \times 5.76) \times 30 = 30,347 \text{ W}$$

This is approximately equal to $29,567 \text{ W}$.

It is important to point out that this calculation assumes that the heat transfer surfaces are clean and free from deposits or fouling. When the heat transfer surfaces get dirty, additional resistances to heat transfer must be considered in the calculations. This subject will be explained in more detail in Chap. 5.

Simplified Correlations for Air at Atmospheric Pressure. McAdams⁴ presented simplified correlations that are valid for natural convection in air. These expressions can be obtained by substituting the physical properties of air in the Grashoff and Prandtl numbers so that the coefficient h is merely a function of the temperature difference and the characteristic length. These simplified correlations are shown in Table 4-2. Since they are dimensional equations, the units indicated in the table must be used for the correlations to be valid.

In Table 4-3, the physical properties of air at several temperatures are included to allow calculation of the dimensionless groups.

4-4-3 Combined Coefficient of Convection and Radiation

When a body is immersed in an infinite ambient, superimposed on the convective mechanism there exists radiation heat transfer, so the heat transfer rate is the sum of the contributions of both mechanisms, as was explained in Chap. 2. That is,

$$q = h(T_w - T_o) + \sigma \epsilon (T_w^4 - T_o^4) \tag{4-4-23}$$

In this expression, σ is $5.67 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4)$, and ϵ is the surface emissivity, which depends on the type and condition of the surface. The thermal insulation of process equipment is usually finished with an aluminum or galvanized iron jacket. Table 4-4 indicates the emissivity of commonly used materials.

TABLE 4-2 Simplified Equations for Natural Convection in Air at Atmospheric Pressure and Moderate Temperatures

Geometry	Characteristic Dimension L	Flow Regime	Range of $Gr \cdot Pr$	Heat Transfer Coefficient h_a [W/(m ² · K)]
Vertical plates and cylinders	Height	Laminar	10^4 – 10^9	$h_a = 1.42(\Delta T/L)^{1/4}$ (4-4-16)
		Turbulent	10^9 – 10^{13}	$h_a = 1.31\Delta T^{1/3}$ (4-4-17)
Horizontal cylinders	External diameter	Laminar	10^4 – 10^9	$h_a = 1.32(\Delta T/L)^{1/4}$ (4-4-18)
		Turbulent	10^9 – 10^{13}	$h_a = 1.24\Delta T^{1/3}$ (4-4-19)
Horizontal plates, upper surface cold or lower surface hot	As defined in the text	Laminar	10^5 – 2×10^7	$h_a = 1.32(\Delta T/L)^{1/4}$ (4-4-20)
		Turbulent	2×10^7 – 3×10^{10}	$h_a = 1.52\Delta T^{1/3}$ (4-4-21)
Horizontal plates, upper surface hot or lower surface cold	As defined in the text	Laminar	3×10^5 – 3×10^{10}	$h_a = 0.59(\Delta T/L)^{1/4}$ (4-4-22)

Note: L in meters; ΔT in °C. These correlations can be extended to higher or lower pressures with respect to atmospheric pressure by multiplying by the following factors: $(p/1.033)^{1/2}$ for laminar flow and $(p/1.032)^{2/3}$ for turbulent flow, with p in absolute atmospheres.

TABLE 4-3 Properties of Air

Temperature (°C)	Thermal Conductivity k [W/(m · K)]	$\frac{\rho^2 g \beta}{\mu^2}$ [1/(m ³ · K)]	Kinematic Viscosity μ/ρ (m ² /s)	Prandtl Number
10	0.0250	120.3×10^6	14.4×10^{-6}	0.711
20	0.0257	102.9×10^6	15.3×10^{-6}	0.709
30	0.0264	87.4×10^6	16.2×10^{-6}	0.707
40	0.0272	75.8×10^6	17.2×10^{-6}	0.705
50	0.028	65.7×10^6	18.2×10^{-6}	0.704
60	0.0287	57.0×10^6	19.2×10^{-6}	0.702
70	0.0295	49.4×10^6	20.2×10^{-6}	0.701
80	0.0303	43.1×10^6	21.3×10^{-6}	0.699
90	0.0310	38.1×10^6	22.4×10^{-6}	0.697
100	0.0318	33.7×10^6	23.5×10^{-6}	0.695

TABLE 4-4 Emissivities

Material	ϵ
Aluminum plate	0.1–0.2
Aluminum-based paint	0.3–0.7
Steel plate	0.94–0.97
White paint	0.84–0.92
Asphaltic mastic	0.93

A radiation heat transfer coefficient is sometimes defined as

$$h_R = \frac{\sigma \epsilon (T_w^4 - T_o^4)}{T_w - T_o} \tag{4-4-24}$$

Equation (4-4-23) then can be written as

$$Q = (h + h_R)A(T_w - T_o) \tag{4-4-25}$$

4-4-4 Heat Loss through an Insulated Wall in Natural Convection

In Chap. 3 we studied the equations to calculate the heat loss through an insulated wall. Let's consider a vessel containing a hot liquid whose walls are insulated with mineral wool. The heat loss through the wall can be calculated as

$$q = U(T_i - T_o) \quad (4-4-26)$$

with

$$\frac{1}{U} = \frac{1}{h_i} + \frac{\delta}{k} + \frac{1}{h_o} \quad (4-4-27)$$

In Eq. (4-4-27), h_i is the natural convection coefficient of the internal side (liquid-wall), and h_o is the wall-air coefficient; δ is the insulation thickness, and k is its thermal conductivity (the resistance of the metallic wall was neglected). The calculation is not straightforward because both h_i and h_o depend on the temperature of the walls in contact with the fluids, and these are not known. It is necessary, therefore, to follow an iterative procedure to solve the problem. The proposed methodology is

1. Make a first guess for U using approximate values for the h 's.
2. Calculate q .
3. Estimate the internal and external temperatures of the wall with $\Delta T = q/h$.
4. With these temperatures, calculate the film temperatures and the convection coefficients with the respective correlations.
5. Recalculate U and q with the new coefficients.
6. Recalculate the wall temperatures.
7. Compare with the previous ones and go back to step 4 if necessary.

For a first guess of the heat transfer coefficients, the following values are suggested:

For water and aqueous solutions	1,000 W/(m ² · K)
For light organic liquids	350 W/(m ² · K)
For heavy organic liquids	75 W/(m ² · K)
For gases (combined convection radiation coefficient)	10 W/(m ² · K)
For steam as the heating medium	8,500 W/(m ² · K)

Example 4.4 Calculate the heat loss of a vertical cylinder tank with 20-m diameter and 10-m height insulated with 1½-in mineral wool in the vertical walls. The tank roof is not insulated. The tank contains a product whose properties are given in the following table and has to be maintained at a minimum temperature of 22°C when the external temperature is 0°C. It can be considered that the tank is full with liquid to near the roof height. The roof is in contact with the internal vapor atmosphere. The volatility of the product is low, so this internal atmosphere is mainly air. The thermal conductivity of the insulation is 0.048 W/(m · K).

Liquid Properties

Temperature (°C)	22	10
ρ (kg/m ³)	802.4	809.7
μ (cP)	18	27
c [J/(kg · K)]	2,260	2,115
k [W/(m · K)]	0.148	0.148

Solution We shall first calculate the thermal expansion coefficient β using density data:

$$\beta = -\frac{1}{\rho} \frac{\Delta\rho}{\Delta T} = \frac{1}{806} \frac{809.7 - 802.4}{12} = 7.5 \times 10^{-4} \text{ K}^{-1} \quad (\text{to be assumed as constant})$$

We shall calculate the heat loss by natural convection and radiation from the vertical wall and roof next:
Vertical wall

- Calling T_1 : liquid temperature (22°C)
 T_2 : metallic wall temperature
 T_3 : external temperature of the insulation
 T_4 : air temperature (0°C)
 h_i : internal heat transfer coefficient
 h_o : external convection-radiation combined coefficient

We shall first calculate an overall heat transfer coefficient using the estimated h 's mentioned in the preceding section. This will be

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + \frac{\Delta x}{k_{\text{ins}}} = \frac{1}{75} + \frac{1}{10} + \frac{0.038}{0.048} = 0.905 \text{ (m}^2 \cdot \text{K)/W}$$

With this coefficient, the heat loss through the wall would be

$$q = U(T_1 - T_4) = (22 - 0)/0.905 = 24.3 \text{ W/m}^2$$

And temperatures T_2 and T_3 may be calculated as

$$T_2 = T_1 - \frac{q}{h_i} = 22 - \frac{24.3}{75} = 21.6$$

$$T_3 = T_4 + \frac{q}{h_o} = 0 + \frac{24.3}{10} = 2.4$$

The film temperatures for calculation of the physical properties are

For air: $(2.4 + 0)/2 = 1.2$

For the liquid: $(22 + 21.6)/2 = 21.8$

And the properties are

	Liquid	Air
Temp (°C)	21.6	1.2
ρ (kg/m ³)	802.4	1.33
μ (cP)	18	0.018
c [J/(kg · K)]	2,260	965
k [W/(m · K)]	0.148	0.024
Pr	274	0.71
ΔT	0.4	2.4
L	10	10
Gr	5.84E+9	4.69E11
Gr · Pr	1.6E12	3.34E11
$h_{\text{conv}} = 0.021k/L(\text{Gr} \cdot \text{Pr})^{0.4}$	23.6	2.08

The radiation coefficient for the external wall is

$$h_R = 5.67 \times 10^{-8} \varepsilon (T_3^4 - T_4^4) / (T_3 - T_4) = 5.67 \times 10^{-8} \times 0.2 (275.4^4 - 273^4) / 2.4 = 0.93 \text{ W}/(\text{m}^2 \cdot \text{K})$$

The external combined coefficient then is $2.08 + 0.93 = 3.01 \text{ W}/(\text{m}^2 \cdot \text{K})$.

Calculating U again, we get

$$\frac{1}{U} = \frac{1}{3.01} + \frac{1}{23.6} + \frac{0.038}{0.048} = 1.15 \text{ (m}^2 \cdot \text{K)}/\text{W}$$

Then

$$q = U \Delta T = \frac{22}{1.15} = 19.1 \text{ W}/\text{m}^2$$

And recalculating the wall temperatures, we get

$$T_2 = T_1 - \frac{q}{h_i} = 22 - \frac{19.1}{28} = 22.6$$

$$T_3 = T_4 + \frac{q}{h_o} = 0 + \frac{19.1}{3.01} = 6.3$$

Since these temperatures do not differ too much from the previous estimation, the physical properties of the fluids will be the same. Recalculating the coefficients:

	Liquid	Air
Temp (°C)	21.6	1.2
ρ (kg/m ³)	802.4	1.33
μ (cP)	18	0.018
c [J/(kg · K)]	2,260	965
k [W/(m · K)]	0.148	0.024
Pr	274	0.71
ΔT	0.4	6.3
L	10	10
Gr	5.84E+9	1.19E12
Gr · Pr	1.6E12	8.47E11
β		0.00364
$h_{\text{conv}} = 0.021k/L (\text{Gr} \cdot \text{Pr})^{0.4}$	23.6	3.03

The radiation coefficient for the external wall results:

$$h_R = 5.67 \times 10^{-8} \varepsilon (T_3^4 - T_4^4) / (T_3 - T_4) = 5.67 \times 10^{-8} \times 0.2 (279.3^4 - 273^4) / 6.3 = 0.95 \text{ W}/(\text{m}^2 \cdot \text{K})$$

The external combined coefficient then is $3.03 + 0.95 = 3.98 \text{ W}/(\text{m}^2 \cdot \text{K})$.

Recalculating U , we get

$$\frac{1}{U} = \frac{1}{3.98} + \frac{1}{23.6} + \frac{0.038}{0.048} = 1.08 \text{ (m}^2 \cdot \text{K)}/\text{W}$$