M1 Heat Transfer to Finned Tubes

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Heat release and absorption from surfaces can be enhanced by fins. The fins are to be placed on the side of poorer heat transfer. Fins will be effective all the more as the ratio of the heat transfer coefficient from the side of better heat transfer to that of poorer increases.

Basic requirement for the following approach is an ideal contact between the fin base and tube or surface. The shown method has to be regarded as a first approach. It will not fit for abnormal dimensions or extremely high Reynolds numbers. Indispensable for such cases is the examination of appropriate literature and far reaching modeling related to experimental data.

A mean heat transfer coefficient has to be found and evaluated on the base of geometrically determined consideration. Therefore, the model cannot differentiate local changes in heat flux due to different temperatures over the fin caused by convection.

It is assumed that the direction of fluid flow corresponds to the orientation of the fins. Heat transfer by thermal radiation will not be considered as well as the heat transfer on the fin's tip which normally contributes less to the total surface.

Design, shape dimensions, and abbreviations of finned surfaces are shown in Fig. 1.

1 Heat Transfer for Finned Tube

The heat flow from a finned tube is

$$\dot{Q} = kA\Delta\theta_{\mathrm{LM}}. \tag{1}$$

Related to the total surface and the temperature gradient between both fluids the heat transfer coefficient k is given by

$$\frac{1}{k} = \frac{1}{\alpha_{\mathrm{v}}} + \frac{A}{A_{\mathrm{i}}}\left(\frac{1}{\alpha_{\mathrm{i}}} + \frac{d_0 - d_{\mathrm{i}}}{2\lambda_{\mathrm{t}}}\right). \tag{2}$$

Equation (2) does not include additional heat transfer resistances caused by

Fouling processes on the fin surface
Poor contact between the tube and fin base

α_{v} is a virtual heat transfer coefficient. Assuming uniform heat transfer coefficient α_{m} for bare tube and fin surface α_{v} is derived from the fin efficiency η_{f}.

The heat flow then is

$$\dot{Q} = \alpha_{\mathrm{m}}(A_{\mathrm{t}} + \eta_{\mathrm{f}}A_{\mathrm{f}})(\Theta_{\mathrm{t}} - \Theta_{\mathrm{a}}). \tag{3}$$

The driving temperature gradient for this case is the difference between surface temperature of the tube and fluid. α_{m} may be calculated as shown in Chap. G6. The characteristic length of the circular fin tube is [1–4]

$$l = \frac{\pi}{2}\sqrt{d_0^2 + h_{\mathrm{f}}^2}$$

Values calculated in this way are greater than those to get with the following method.

Experimentally based values for α_{m} can be derived from Eqs. (15), (16), and (18) for corresponding applications. Characteristic length has to be always the same for the whole calculation procedure. The velocity w_{s} in the smallest cross section is calculated from die velocity in the free flow w_0, the ratio from inflow cross sectional area A_0, and smallest sectional area between fins A_{s}.

$$w_{\mathrm{s}} = w_0\frac{A_0}{A_{\mathrm{s}}}. \tag{4}$$

This value can be fitted furthermore for the velocity change due to changing density with temperature of the fluid.

The fin efficiency is the ratio of the mean temperatures between the respective base of fin or tube and fluid.

$$\eta_{\mathrm{f}} = \frac{\Theta_{\mathrm{f}} - \Theta_{\mathrm{a}}}{\Theta_{\mathrm{t}} - \Theta_{\mathrm{a}}}. \tag{5}$$

Together with this the virtual heat transfer coefficient becomes

$$\alpha_{\mathrm{v}} = \alpha_{\mathrm{m}}\left[\frac{A_{\mathrm{t}}}{A} + \eta_{\mathrm{f}}\frac{A_{\mathrm{f}}}{A}\right] = \alpha_{\mathrm{m}}\left[1 - (1 - \eta_{\mathrm{f}})\frac{A_{\mathrm{f}}}{A}\right]. \tag{6}$$

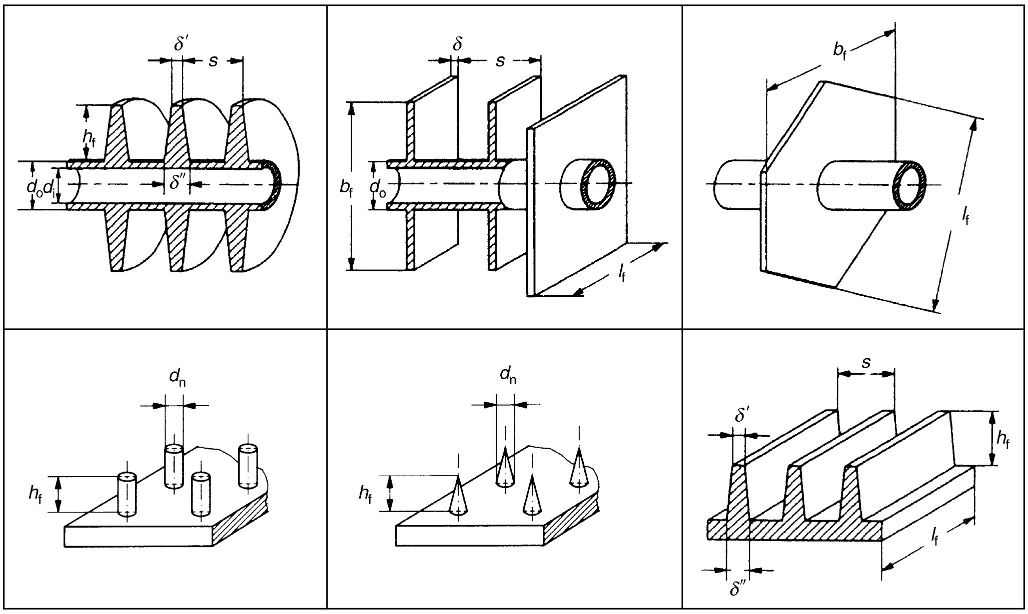
The formal way to calculate fin efficiency is

$$\eta_{\mathrm{f}} = \frac{\tanh X}{X} \tag{7}$$

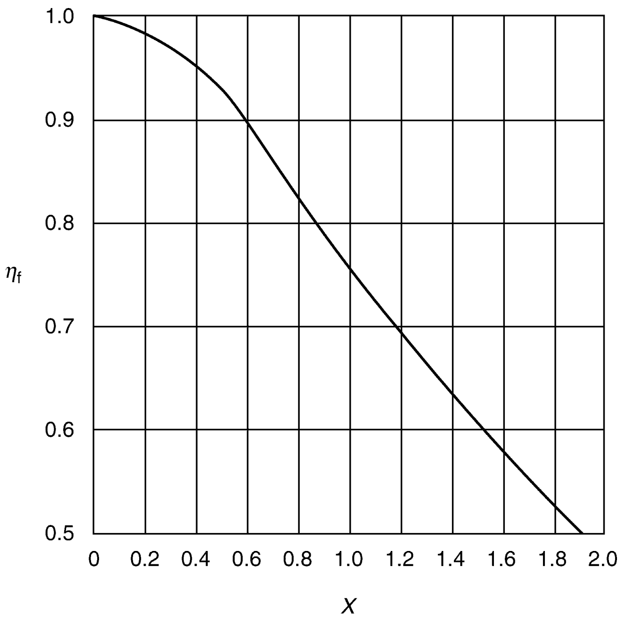
with

$$X = \varphi\frac{d_0}{2}\sqrt{\frac{2\alpha_{\mathrm{m}}}{\lambda_{\mathrm{f}}\delta}}. \tag{8}$$

VDI-GVC (ed.), *VDI Heat Atlas*, DOI 10.1007/978-3-540-77877-6_50,



M1. Fig. 1. Different designs for finned surfaces.



M1. Fig. 2. Efficiency factor of finned surfaces.

In Eq. (8) the product $\varphi d_0/2$ has the meaning of a weighted fin height which incorporates the design δ stays for the fin thickness. The function $\eta_f = f(X)$ is plotted in Fig. 2. The analytical calculation can be done by

$$\eta_f = \frac{\tanh X}{X} = \frac{1}{X}\frac{e^X - e^{-X}}{e^X + e^{-X}}. \tag{9}$$

Commonly used fins types are discussed next.

2 Examples for Fin Geometry

2.1 Circular Fins

$$\varphi = \left(\frac{D}{d_0} - 1\right)\left[1 + 0.35\ln\left(\frac{D}{d_0}\right)\right]. \tag{10}$$

For conic fins with thickness δ'' at the base and δ' at the tip, the mean δ is defined by

$$\delta = \frac{1}{2}(\delta'' + \delta'). \tag{11}$$

2.2 Rectangular Fins

$$\varphi = (\varphi' - 1)(1 + 0.35 \ln \varphi'), \tag{12}$$

$$\varphi' = 1.28\frac{b_f}{d_0}\sqrt{\left(\frac{l_f}{b_f} - 0.2\right)}. \tag{13}$$

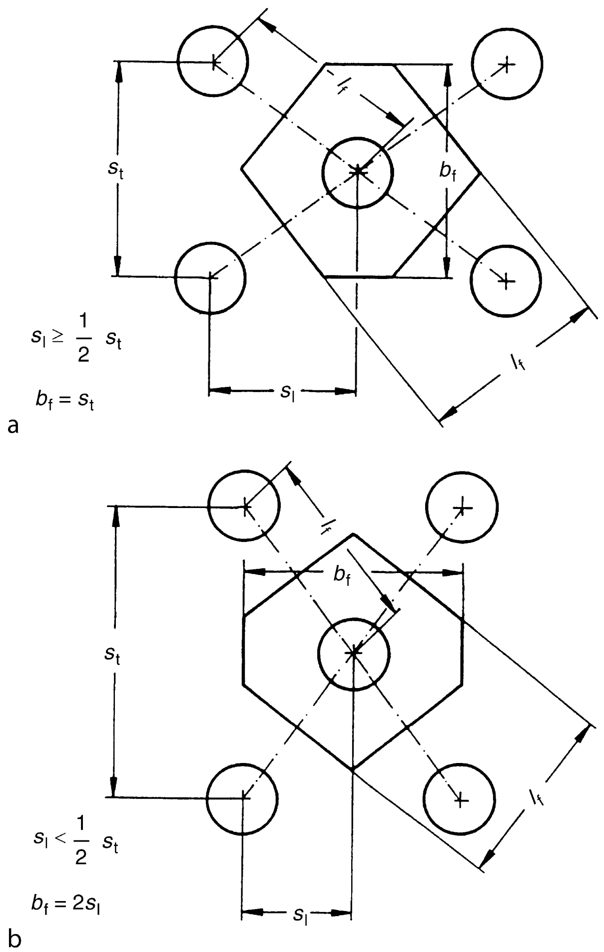
2.3 Adjacent Fins

For arrangements with in-line banks Eqs. (12) and (13) are valid.

For staggered banks a hexagon fin is to be defined for every tube with

$$\varphi' = 1.27\frac{b_f}{d_0}\sqrt{\left(\frac{l_f}{b_f} - 0.3\right)} \quad \text{with } l_f = \sqrt{s_l^2 + \frac{s_t^2}{4}}. \tag{14}$$

Along with the spacing two arrangements are to be distinguished according to Fig. 3 [5].



M1. Fig. 3. Schematic picture of different gaps in the hexagonally designed cross sections between parallel finned tubes in a bundle.

2.4 Straight Fins on Flat Plate

For this case in Eq. (8) the expression $\varphi d_0/2$ has to be substituted by the height of fin h_f and δ becomes

$$\delta = \frac{3}{4}\delta'' + \frac{1}{4}\delta'$$

2.5 Pin or Tip Needles on Flat Plate

Here the expression $\varphi d_0/2$ in Eq. (8) has to be substituted by the height of fin h_f and δ becomes

$$\delta = \frac{1}{2} d_n \quad \text{for pins}$$

$$\delta = \frac{9}{8} d_n \quad \text{for needles}$$

2.6 Heat Transfer in Banks of Finned Tubes in Cross Flow

Heat transfer in banks of finned tubes depends on geometric factors, physical properties, and the velocity of the fluid between the fins and the rows. Inflow pattern and the formation of microturbulences by surface roughness play an important role and are difficult to control. An all-in-one solution for a variety of applications and design cannot be given.

The advice to solve given problems of heat transfer in banks of finned tubes is to look for an empirical calculation formula on the base of experimental data [6–10]. The following equations are derived from industrial data [11] and the comparison with [12, 13].

This leads to more than four rows and

- Inline arrangement

$$\mathrm{Nu_d} = 0.22\mathrm{Re_d^{0.6}}\left(\frac{A}{A_{t0}}\right)^{-0.15}\mathrm{Pr^{!/3}}\,. \qquad (15)$$

- Staggered banks

$$\mathrm{Nu_d} = 0.38\mathrm{Re_d^{0.6}}\left(\frac{A}{A_{t0}}\right)^{-0.15}\mathrm{Pr^{!/3}}\,. \qquad (16)$$

The suffix d prompts to take as characteristic length the outer diameter d_0 of the tube. For circular fins, the ratio A/A_{t0} of the finned surface to the surface of the base tube becomes

$$\frac{A}{A_{t0}} = 1 + 2\frac{h_f(h_f + d_0 + \delta)}{sd_0}\,. \qquad (17)$$

The most important factors in this Eq. (17) are height h and spacing s (Fig. 1). They are essential for the formation of flow between fins and tubes and vary widely in industrial applications.

Calculations with the given Eqs. (15) and (16) were fitted within a range of ±10% to ±25% for $10^3 < \mathrm{Re_d} < 10^5$ and $5 \leq A/A_{t0} \leq 30$

For banks with one to three rows, it is recommended to modify the factor C of the Nusselt power equation

$$\mathrm{Nu_d} = C\mathrm{Re_d^{0.6}}\left(\frac{A}{A_{t0}}\right)^{-0.15}\mathrm{Pr^{!/3}}\,. \qquad (18)$$

For in-line arrangement $C = 0.20$, for staggered arrangement with two rows $C = 0.33$, and for three rows $C = 0.36$.

Example

Air has to be heated up from 90 to 120°C. Heating fluid is vapour condensing at 130°C. The mass flow of air is 1.92 kg/s.

The required power then is 59 kW.

Circular fin tubes	D = 56 mm, d_0 = 25.4 mm, δ = 0.4 mm	
	a = 2.42 mm, d_i = 21 mm	
Material	Aluminum with λ = 209 W/(m K)	
Arrangement	9 fins/in.	
	Spacing fins	s = 2.82 mm
	Spacing transversal	s_t = 60 mm
Inflow cross section	1 m^2	
Inflow velocity	2 m/s, calculated from mass flow and temperature	

Heat transfer coefficient	α_i = 10,454 W/(m^2 K)	
Width heat exchanger	17 tubes parallel -- >	17 s_t = 1.02 m
Height heat exchanger	1 m^2/1.02 m = 0.98 m	
Number fins single tube	0.98 m/0.00282 m = 348 fins	
A_f single fin	$A_f = 2\frac{\pi}{4}(D^2 - d_0^2)$ $= 2\frac{\pi}{4}(56^2 - 25.4^2)10^{-6}\,\text{m}^2$ $= 3.913 \cdot 10^{-3}\,\text{m}^2$	
A_f per tube	$A_f = 348 \cdot 3.913 \cdot 10^{-3}\,\text{m}^2$ $= 1.362\,\text{m}^2$	
A_t per tube	$A_t = (348 + 1)\pi d_0 a$ $= 349 \cdot \pi \cdot 25.4 \cdot 2.42 \cdot 10^{-6}\,\text{m}^2 = 0.067\,\text{m}^2$	
A_{t0} per tube	$A_{t0} = \pi \cdot d_0 \cdot 0.98m$ $= \pi \cdot 25.4 \cdot 10^{-3}\,\text{m} \cdot 0.98\,\text{m} = 0.078\,\text{m}^2$	
A per tube	$A = 1.362\,\text{m}^2 + 0.067\,\text{m}^2 = 1.429\,\text{m}^2$	
A_i per tube	$A_i = 0.98\,\text{m} \cdot \pi \cdot d_i$ $= 0.98\,\text{m} \cdot \pi \cdot 21 \cdot 10^{-3}\,\text{m} = 0.065\,\text{m}^2$	

We assume a heat exchanger with circular fins and in-line arrangement.
Straitened cross section of flow:

$$\frac{A_0}{A_s} = \frac{s_t(a+\delta)}{(s_t - d_0)a + (s_t - D)\delta} = \frac{60 \cdot 2.82}{(60 - 25.4) \cdot 2.42 \cdot (60 - 56) \cdot 0.4} = 1.984.$$

Flow velocity in the smallest cross section:

$$w_s = w_0 \frac{A_0}{A_s} = 2\text{m/s} \cdot 1.984 = 3.97\,\text{m/s}.$$

Influence of temperature:

$$w_{sT} = w_s \frac{273 + \frac{1}{2}(120 + 90)}{273 + 90} = 3.97\ \text{m/s} \cdot 1.04 = 4.13\ \text{m/s}.$$

Ratio of surfaces

$$\frac{A}{A_{t0}} = \frac{1.429\,\text{m}^2}{0.078\,\text{m}^2} = 18.321\ (\text{note: Eq. (17) will give } 18.558).$$

Heat transfer air side

$$\text{Re}_d = \frac{d_0 w_{sT} \rho_{air}}{\eta_{air}} = \frac{0.0254\,\text{m} \cdot 4.13\,\text{m/s} \cdot 0.909\,\text{kg/m}^3}{22.37 \cdot 10^{-6}\,\text{kg/(ms)}} = 4,236,$$

$$\text{Nu}_d = 0.22\text{Re}_d^{0.6}\left(\frac{A}{A_{to}}\right)^{-0.15} \cdot \text{Pr}^{1/3}$$
$$= 0.22 \cdot 4263^{0.6} \cdot 18.321^{-0.15} \cdot 0.706^{1/3} = 19.07,$$

$$\alpha_m = \frac{\text{Nu}_d \lambda_{air}}{d_0} = \frac{19.07 \cdot 0.0321\,\text{W/(m K)}}{0.0254\,\text{m}} = 24.10\,\text{W/(m}^2\text{K)}.$$

Fin efficiency: Circular fins in-line Eq. (10)

$$\varphi = \left(\frac{D}{d_0} - 1\right)\left[1 + 0.35\ \ln\left(\frac{D}{d_0}\right)\right] = 1.54,$$

$$X = \varphi \frac{d_0}{2}\sqrt{\frac{2\alpha_m}{\lambda_f \delta}} = 1.54 \frac{0.0254\,\text{m}}{2}\sqrt{\frac{2 \cdot 24.14\,\text{W/(m}^2\text{K)}}{209\ \text{W/(mK)} \cdot 0.0004\,\text{m}}} = 0.47,$$

$$\eta_f = \frac{\tanh X}{X} = \frac{1}{X}\frac{e^X - e^{-X}}{e^X + e^{-X}} = \frac{1}{0.47}\frac{e^{0.47} - e^{-0.47}}{e^{0.47} + e^{-0.47}} = 0.93,$$

$$\alpha_v = \alpha_m\left[1 - (1 - \eta_f)\frac{A_f}{A}\right]$$
$$= 24.10\left[1 - (1 - 0.93)\frac{1.362}{1.429}\right]\text{W/(m}^2\text{K)}$$
$$= 22.49\,\text{W/(m}^2\text{K)}$$

$$\frac{1}{k} = \frac{1}{\alpha_v} + \frac{A}{A_i}\left(\frac{1}{\alpha_i} + \frac{d_0 - d_i}{2\lambda_t}\right)$$
$$= \left[\frac{1}{22.49} + \frac{1.429}{0.065}\left(\frac{1}{10{,}454} + \frac{0.0254 - 0.021}{2 \cdot 209}\right)\right]\text{m}^2\text{K/W}$$
$$= 0.0468\,\text{m}^2\text{K/W},$$
$$k = 21.37\,\text{W/m}^2\text{K}.$$

Logarithmic mean temperature difference

$$\Delta\Theta_{LM} = \frac{\Theta_{out} - \Theta_{in}}{\ln\frac{\Theta_{tube} - \Theta_{in}}{\Theta_{tube} - \Theta_{out}}} = \frac{30}{\ln\frac{40}{10}} = 21.64\,\text{K}.$$

Heat transfer surface necessary

$$A = \frac{\dot{Q}}{k\Delta\Theta_{LM}} = \frac{59{,}000}{21.37 \cdot 21.64}\ \text{m}^2 = 127.58\,\text{m}^2.$$

Number of rows

$$\text{NR} = \frac{127\,\text{m}^2}{17 \cdot 1.429\,\text{m}^2} = 5.25 \rightarrow 6\ \text{rows}.$$

Six rows are chosen.

A	total outer surface
A_s	smallest cross-sectional area
A_t	free outer surface of tube
A_{to}	surface of bare tube without fins
A_i	inside surface of tubes
A_f	fin surface
A_0	inflow cross-sectional area
X	operand Eq. (8)
a	free space between fins
b_f	width of angular fin
D	outer diameter of fin
d_o	outer diameter of tube
d_i	inner diameter of tube
d_n	diameter of needles
h_f	height of fin
k	overall heat transfer coefficient

l_f	length of angular fin
$\dot{Q}$	heat flow
δ	thickness of fin
s_l	spacing longitudinal
s_t	spacing transversal
w_s	velocity in the smallest cross section
w_0	inflow velocity
α_i	heat transfer coefficient in the inner tube
α_m	mean heat transfer coefficient for tube and fin
α_v	virtual heat transfer coefficient
$\Delta\Theta_{LM}$	logarithmic mean temperature difference
Θ_t	surface temperature tube
Θ_f	surface temperature fin
Θ_a	ambient temperature
η_f	fin efficiency
λ	thermal conductivity, uniform for fin f or tube t
ρ	density fluid
φ	operand Eq. (8)

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